Simplification & Approximation

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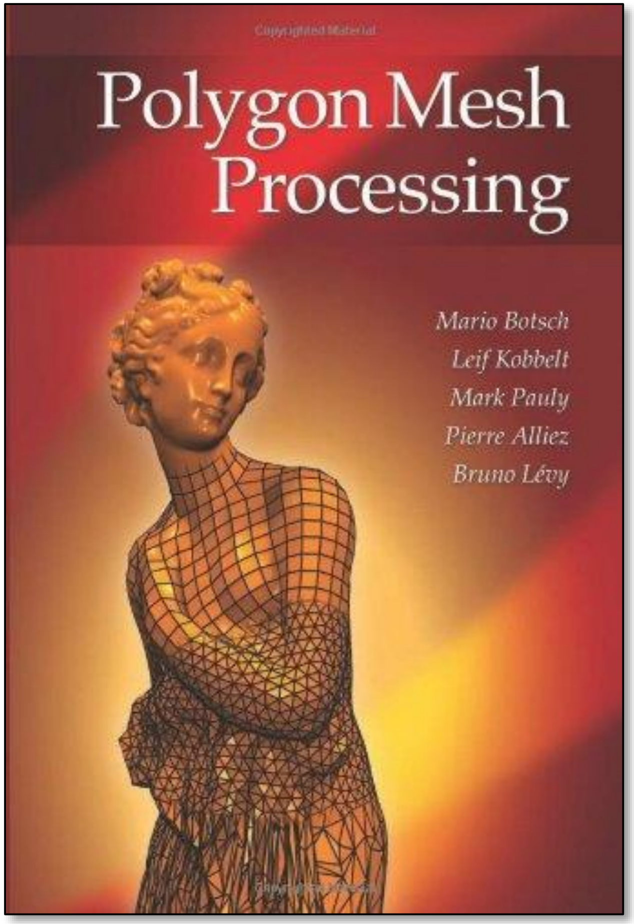
informatics mathematics

European Research Council

Outline

- Motivations
- Simplification
- Approximation
- Guarantees
- Remaining Challenges

Material from…



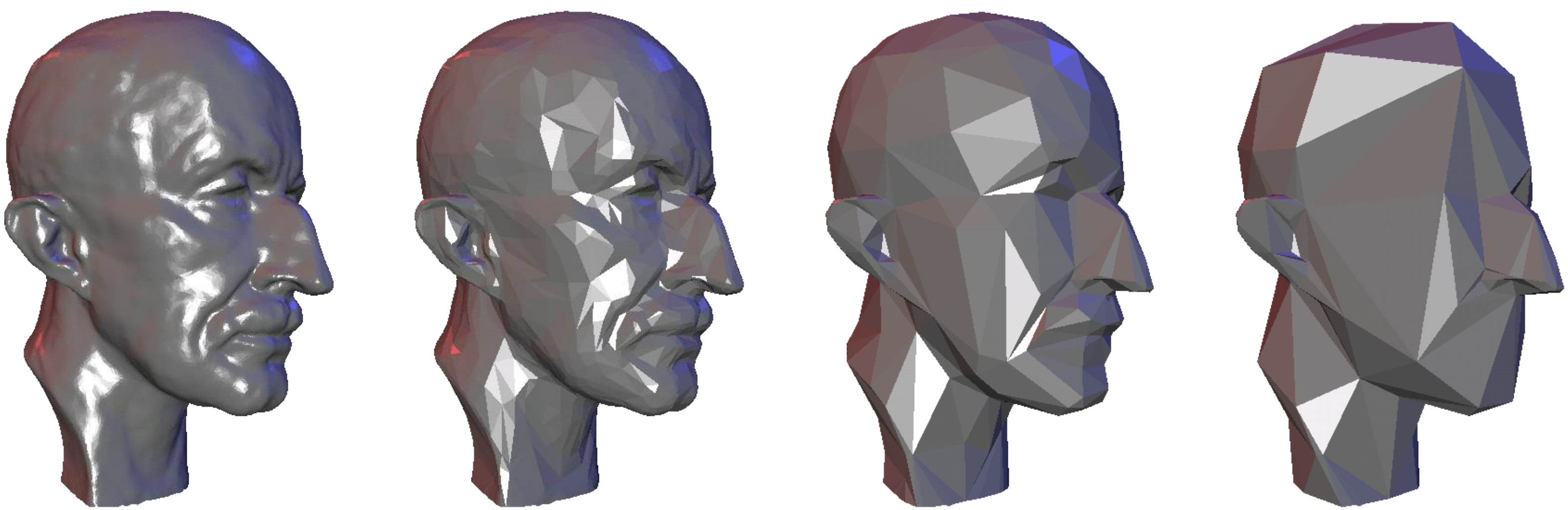
- Mario Botsch

and…

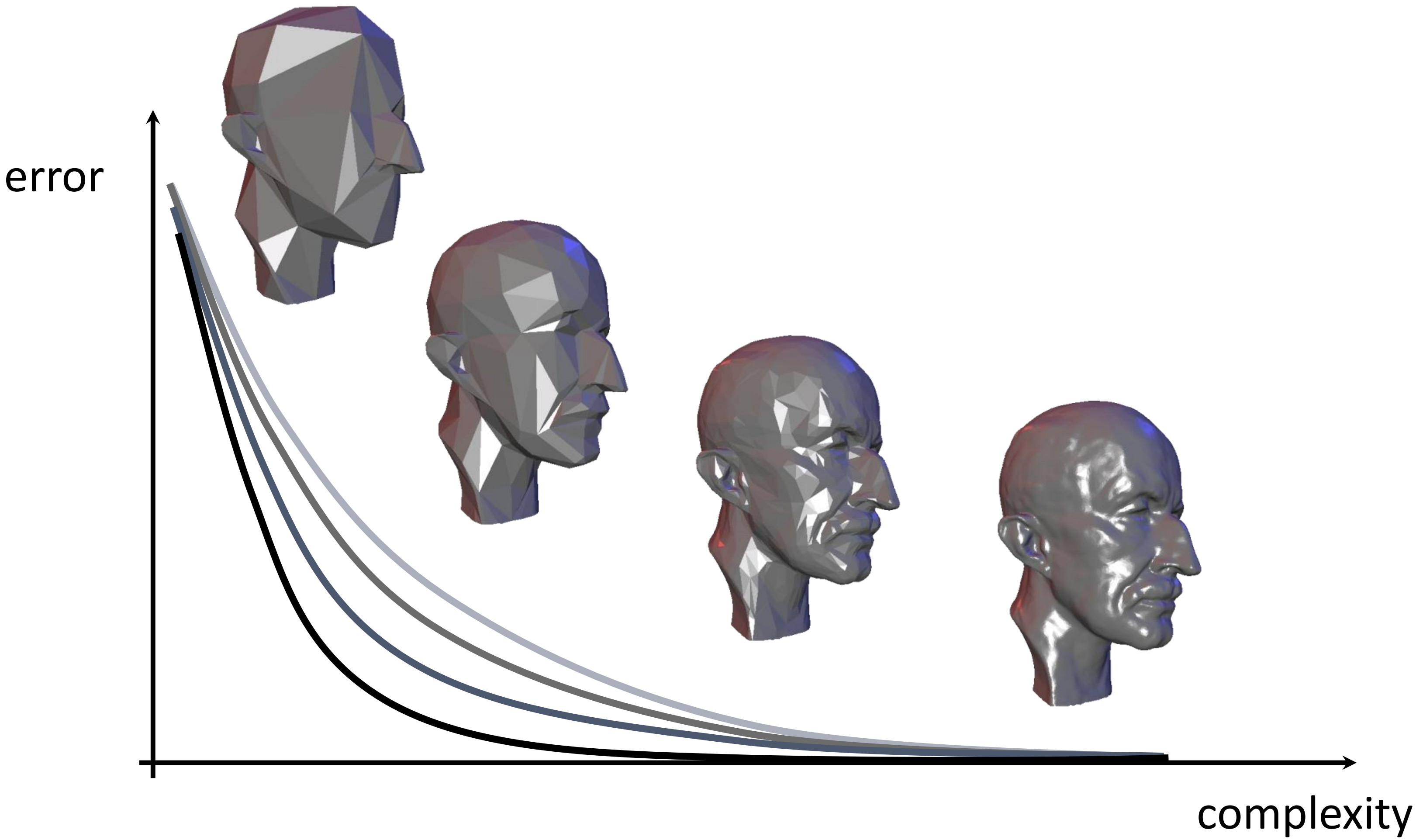
- David Cohen-Steiner
- Mathieu Desbrun
- Florent Lafarge
- Manish Mandad
- …

Motivations

- Multi-resolution hierarchies for
  - efficient geometry processing
  - level-of-detail (LOD) rendering



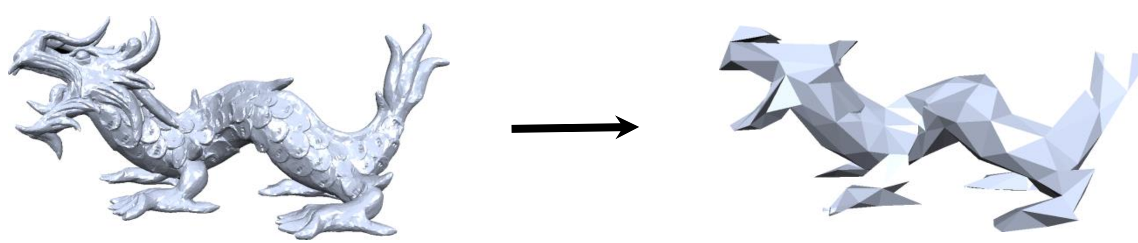
Complexity-Error Tradeoff



Problem Statement

- Given: $\mathcal{M} = (\mathcal{V}, \mathcal{F})$
- Find: $\mathcal{M}' = (\mathcal{V}', \mathcal{F}')$ such that

1. $|\mathcal{V}'| = n < |\mathcal{V}|$ and $\|\mathcal{M} - \mathcal{M}'\|$ is minimal, or
2. $\|\mathcal{M} - \mathcal{M}'\| < \epsilon$ and $|\mathcal{V}'|$ is minimal



$\mathcal{M}$ $\qquad\qquad$ $\mathcal{M}'$

Problem Statement

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hard! [Agarwal-Suri 1998]

→ look for sub-optimal solution

Problem Statement

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2. $\|\mathcal{M} - \mathcal{M}'\| < \epsilon$ and $|\mathcal{V}'|$ is minimal

- Additional fairness criteria
  - normals, triangle shape, appearance attributes, ...

Simplification

Simplification

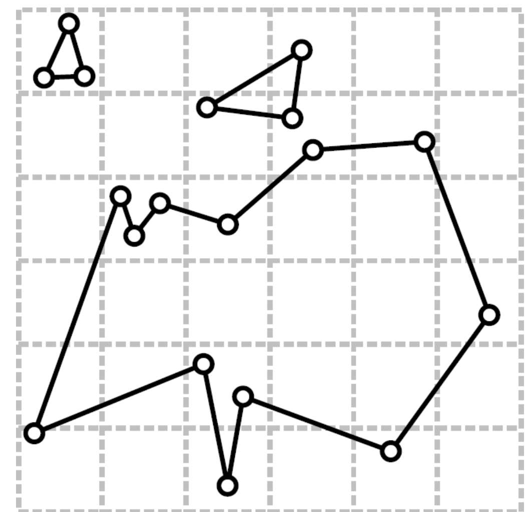
- Vertex Clustering
- Iterative Decimation
- Extensions

Simplification

- **Vertex Clustering**
- Iterative Decimation
- Extensions

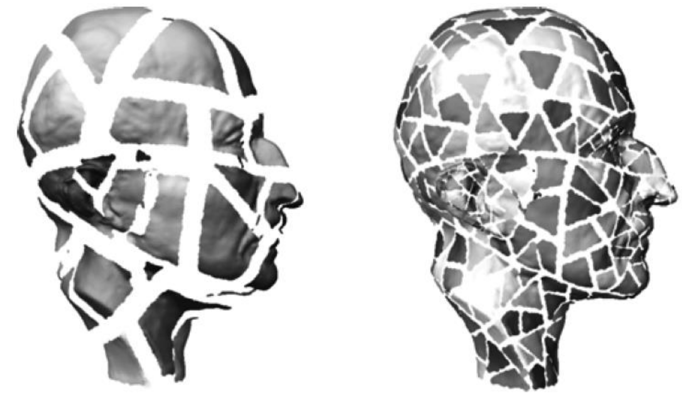
Vertex Clustering

- Cluster Generation
  - Uniform 3D grid
  - Map vertices to cluster cells
- Computing a representative
- Mesh generation
- Topology changes



Vertex Clustering

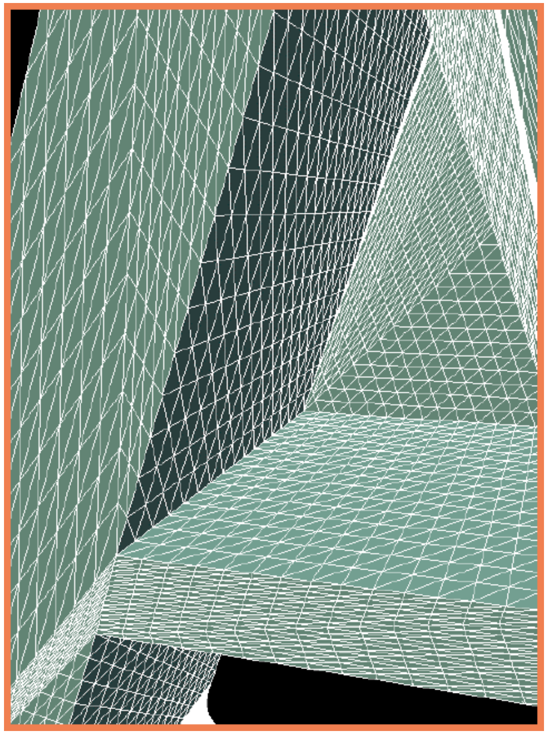
- Cluster Generation
  - Hierarchical approach
  - Top-down or bottom-up
- Computing a representative
- Mesh generation
- Topology changes

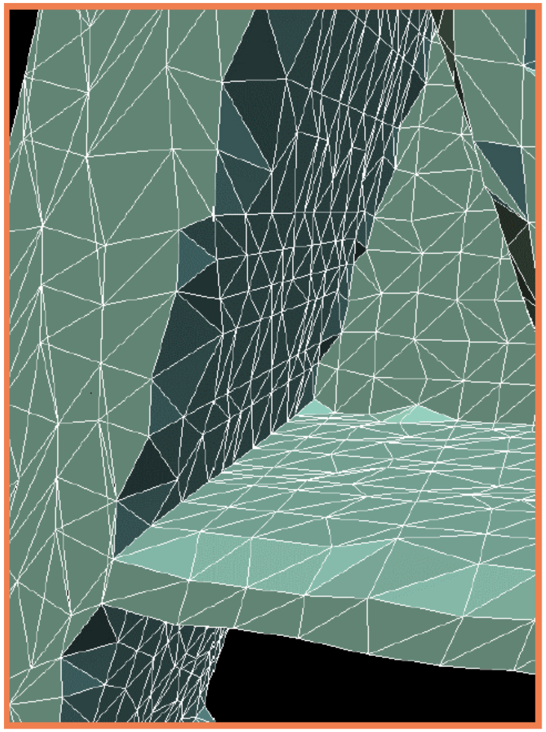


Vertex Clustering

- Cluster Generation
- Computing a representative
  - Average/median vertex position
  - Error quadrics
- Mesh generation
- Topology changes

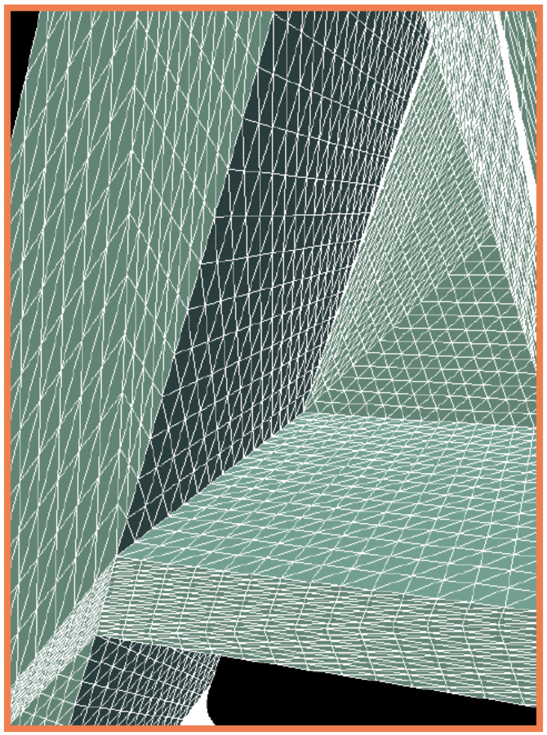
Computing a Representative

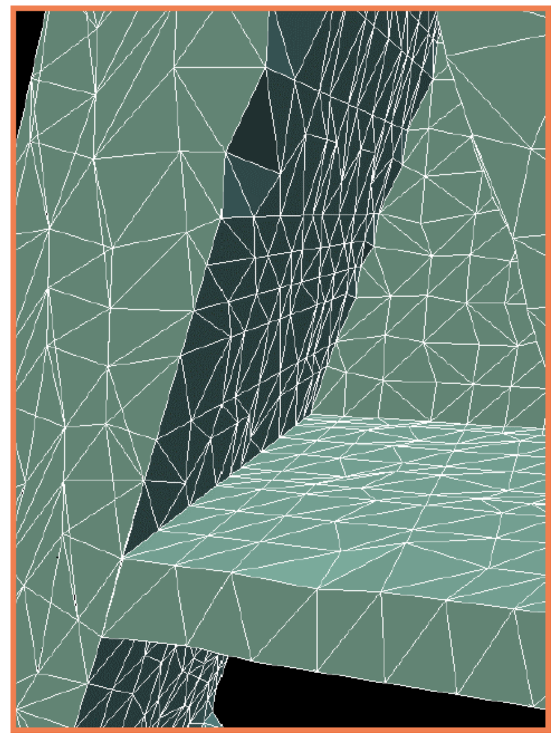




- Average vertex position → Low-pass filter

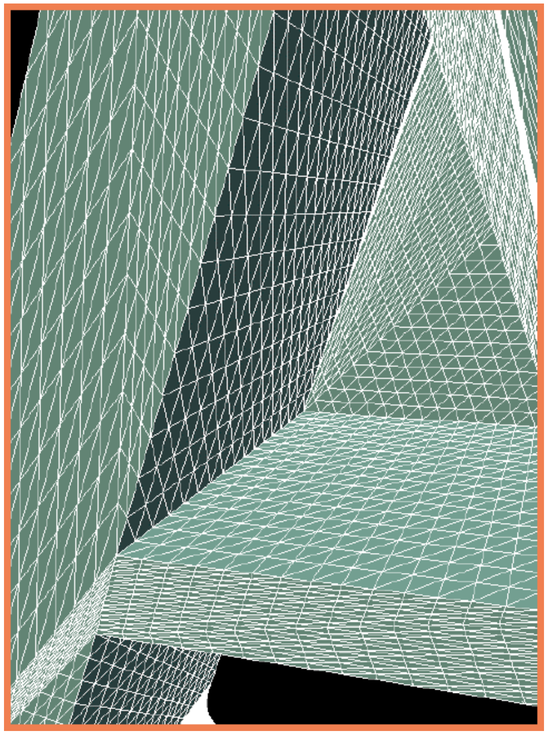
Computing a Representative

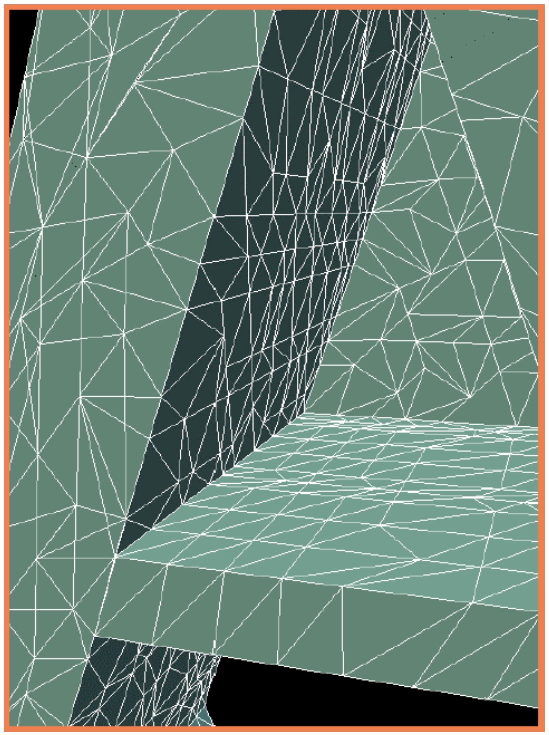




- Median vertex position → Sub-sampling

Computing a Representative





- Error quadrics

Error Quadrics

- Squared distance to plane

$$p = (x, y, z, 1)^T, \;\; q = (a, b, c, d)^T$$

$$dist(q,p)^2 = (q^T p)^2$$

$$Q_q = \begin{bmatrix} a^2 & ab & ac & ad \\ ab & b^2 & bc & bd \\ ac & bc & b^2 & cd \\ ad & bd & cd & d^2 \end{bmatrix}$$

Error Quadrics

- Sum distances to vertex' planes

$$\sum_i dist(q_i, p)^2$$

- Point location that minimizes the error

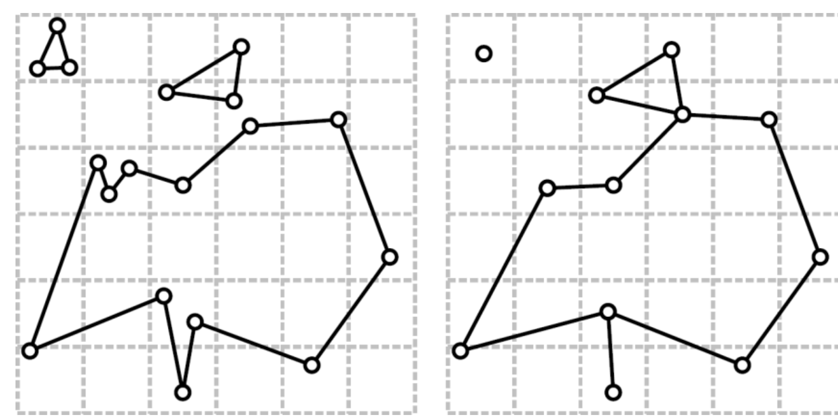
$$\begin{bmatrix} q_{11} & q_{12} & q_{13} & q_{14} \\ q_{21} & q_{22} & q_{23} & q_{24} \\ q_{31} & q_{32} & q_{33} & q_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} p^* = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
  - Clusters $p\{p_0,\ldots,p_n\}$, $q\{q_0,\ldots,q_m\}$
  - Connect (p,q) if there was an edge (p_i,q_j)
- Topology changes

Vertex Clustering

- Cluster Generation
- Computing a representative
- Mesh generation
- Topology changes
  - If different sheets pass through one cell
  - Not manifold



Simplification

- Vertex Clustering
- **Iterative Decimation**
- Extensions

Iterative Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes

General Setup

Repeat:

- pick mesh region
- apply decimation operator

Until no further reduction possible

Greedy Optimization

For each region

- evaluate quality after decimation
- enqueue(quality, region)

Repeat:

- pick best mesh region
- apply decimation operator
- update queue

Until no further reduction possible

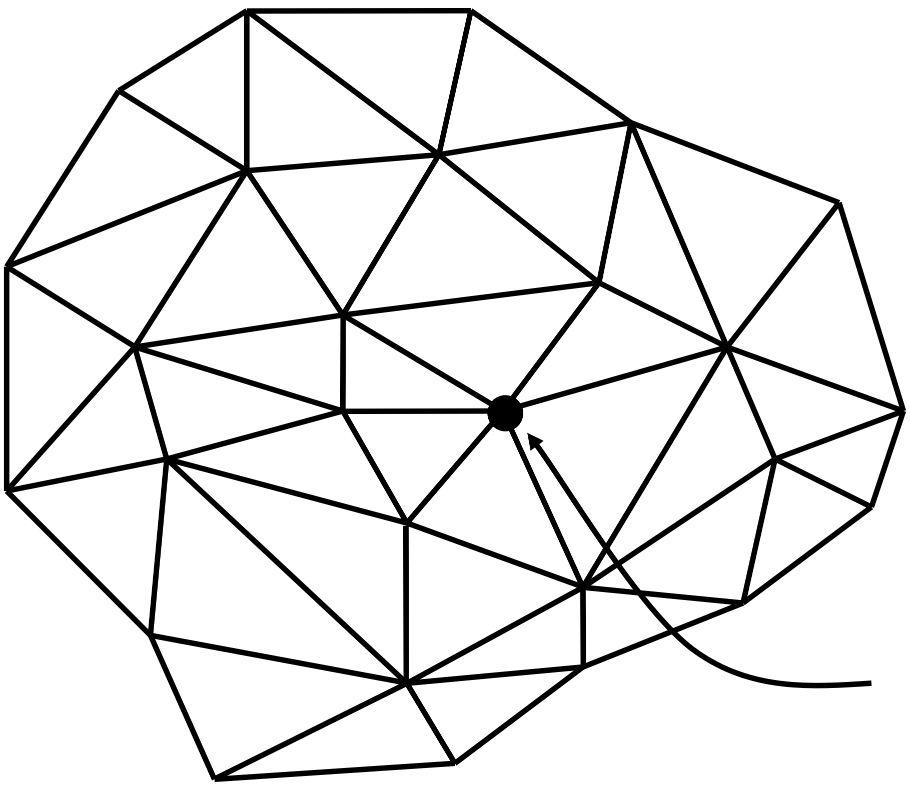
Iterative Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes

Decimation Operators

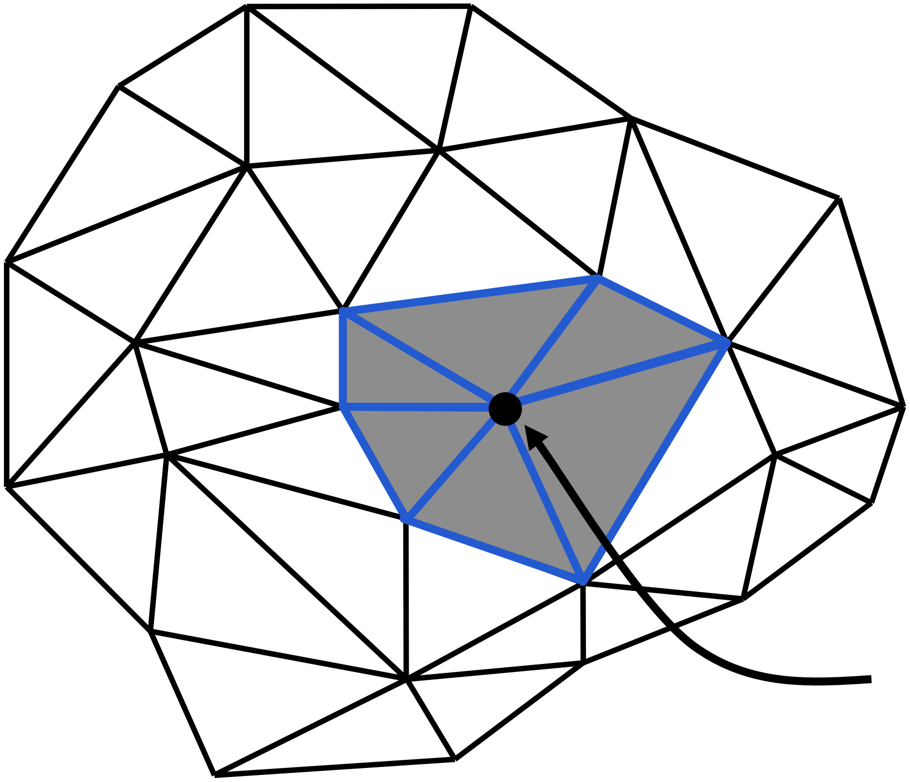
- What is a "region" ?
- What are the DOF for re-triangulation?
- Classification
  - Topology-changing vs. topology-preserving
  - Subsampling vs. filtering

Vertex Removal



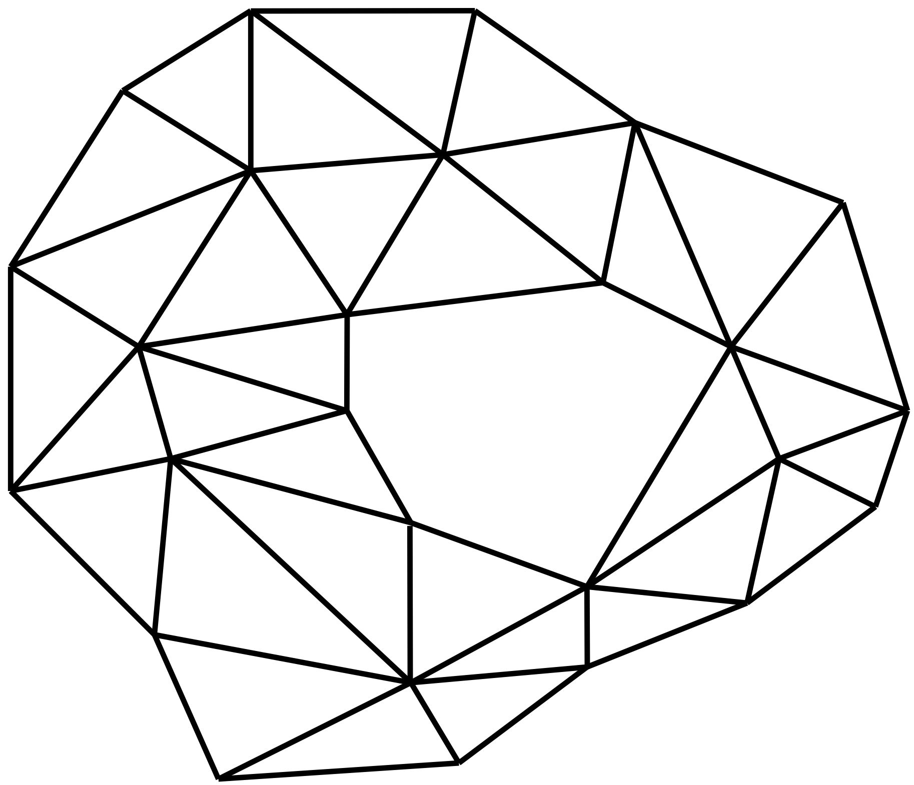
Select a vertex to be eliminated

Vertex Removal



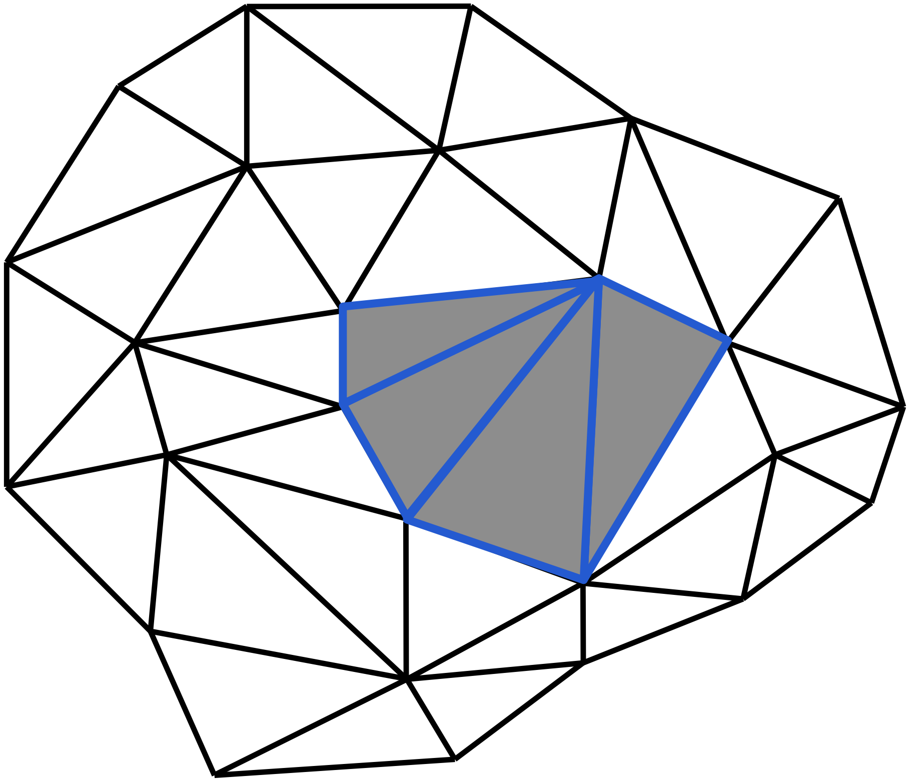
Select all triangles
sharing this vertex

Vertex Removal



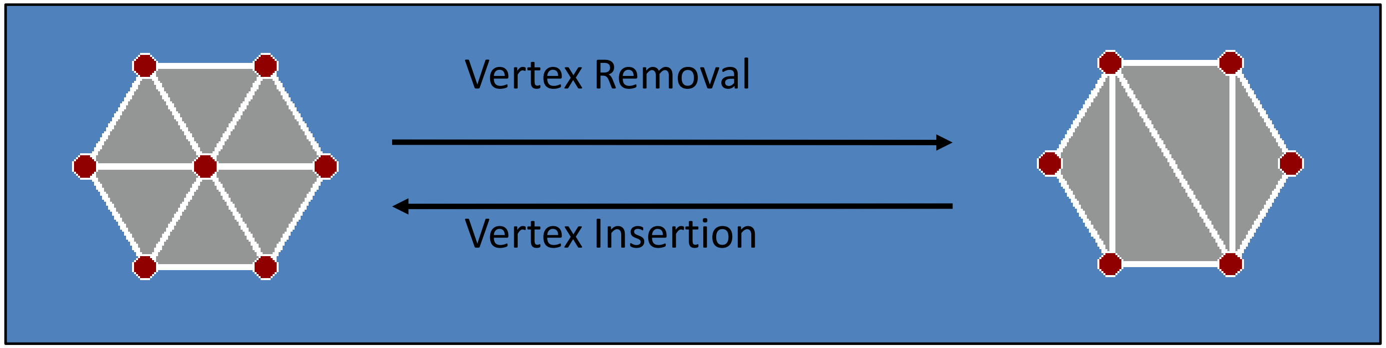
Remove the selected triangles, creating the hole

Vertex Removal



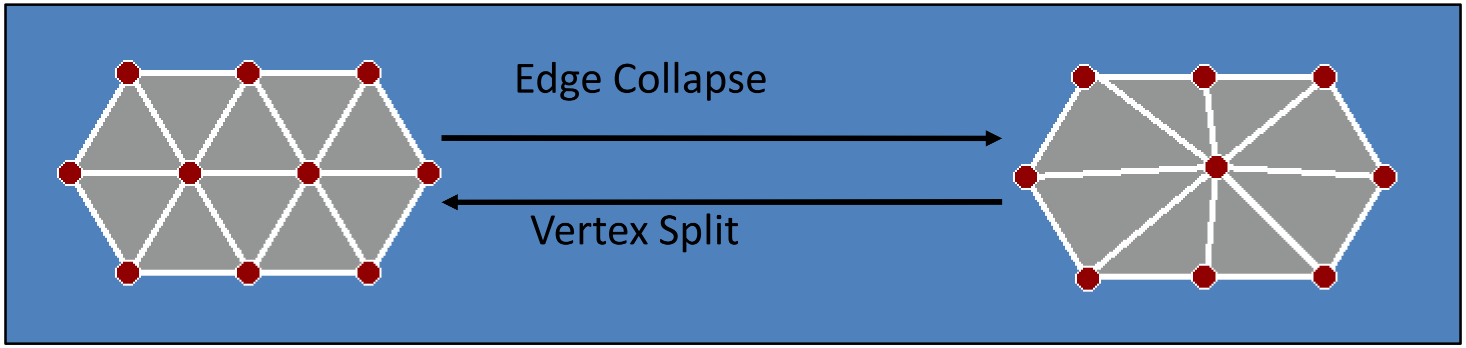
Fill the hole with triangles

Decimation Operators



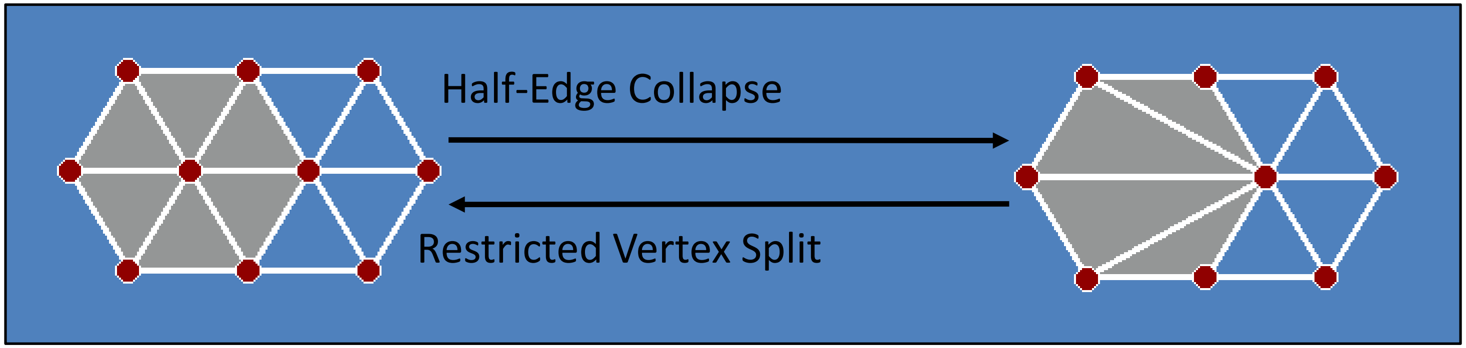
- Remove vertex
- Re-triangulate hole
  - Combinatorial DOFs
  - Sub-sampling

Decimation Operators



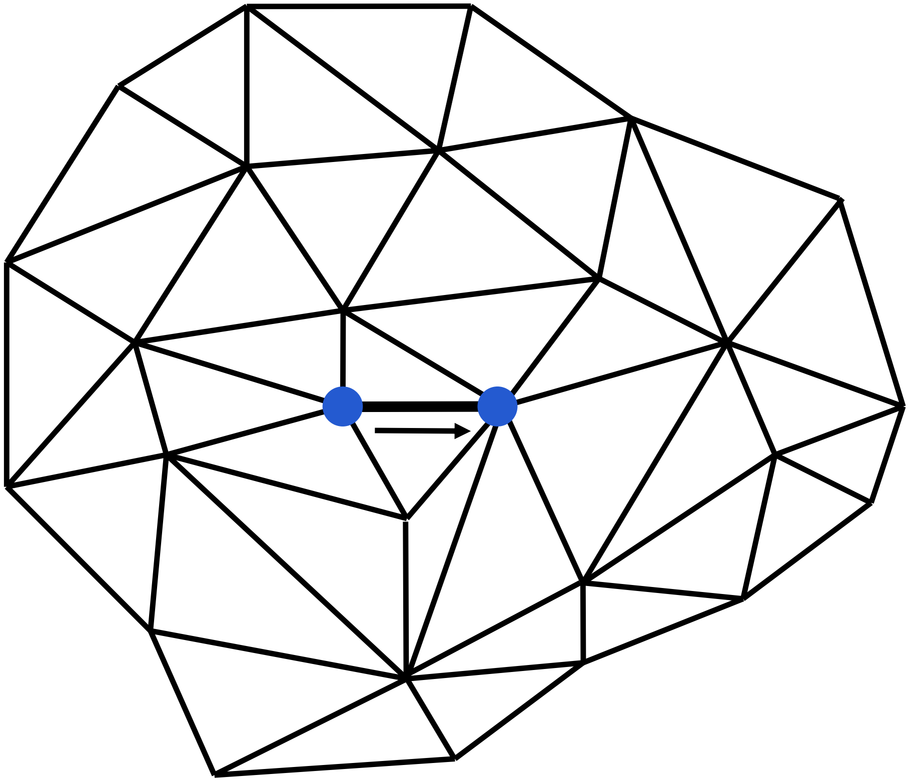
- Merge two adjacent triangles
- Define new vertex position
  - Continuous DOF
  - Filtering

Decimation Operators

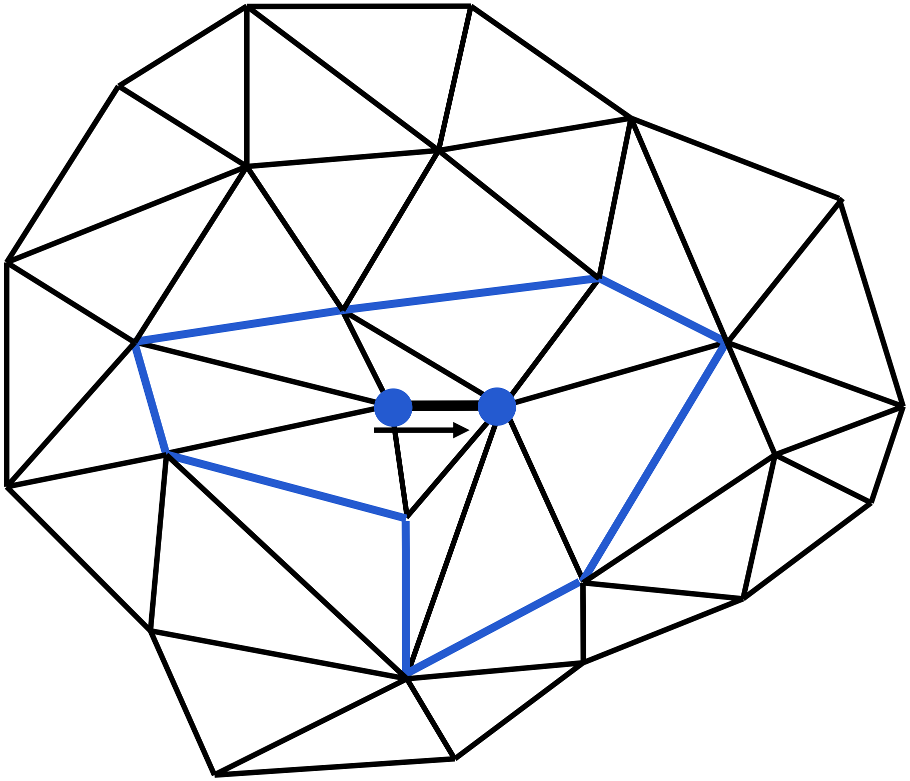


- Collapse edge into one end point
  - Special vertex removal
  - Special edge collapse
- No DOFs
  - One operator per half-edge
  - Sub-sampling

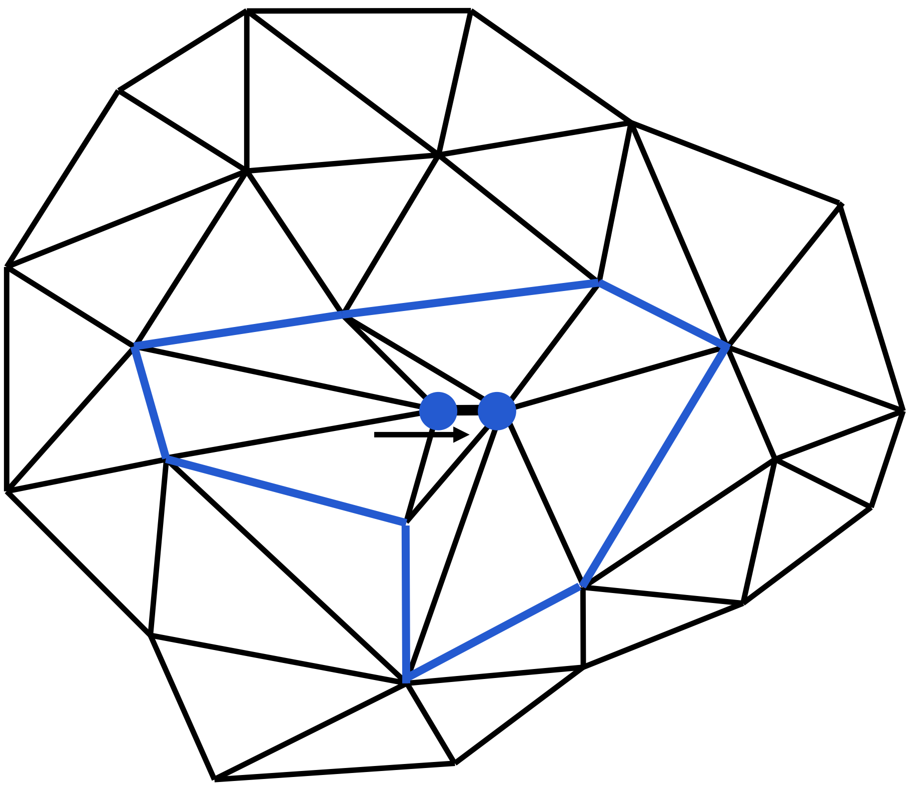
Edge Collapse



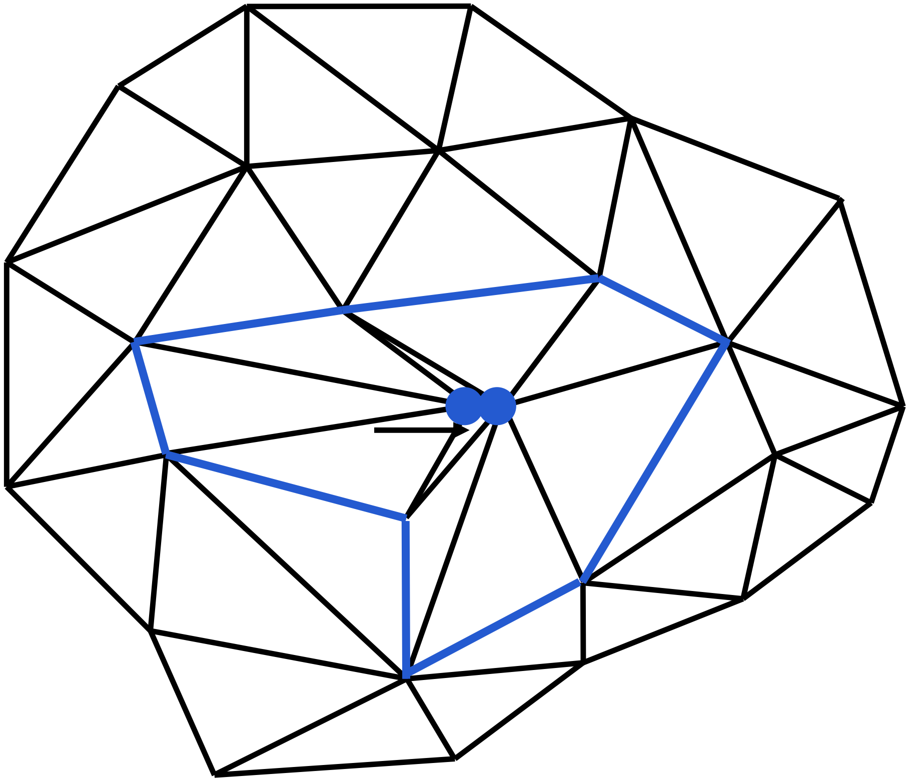
Edge Collapse



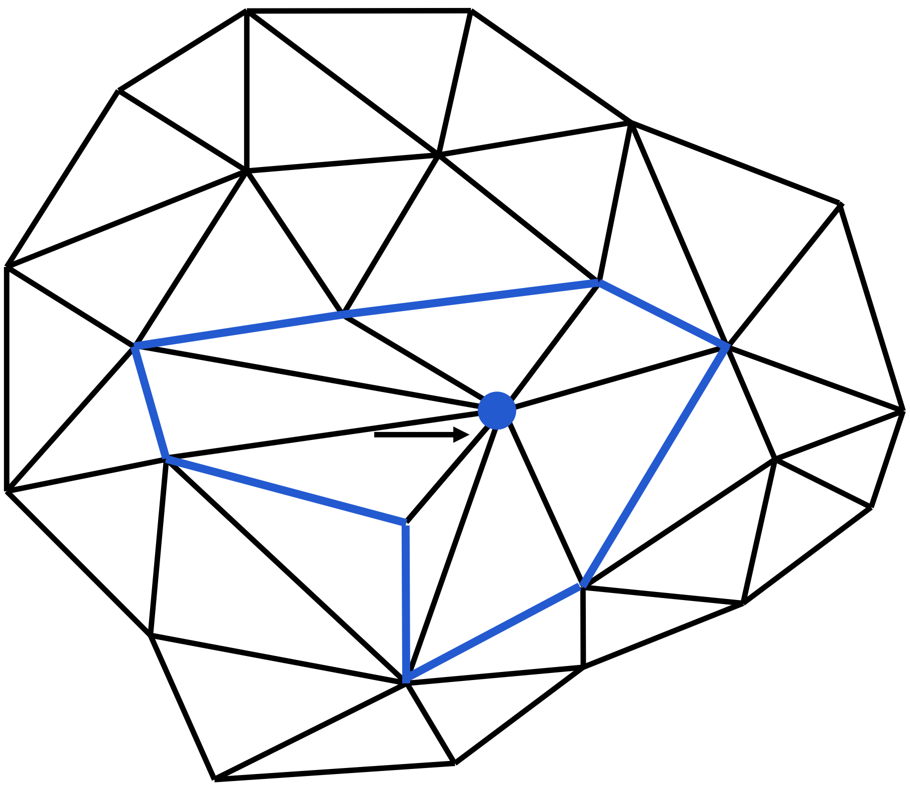
Edge Collapse



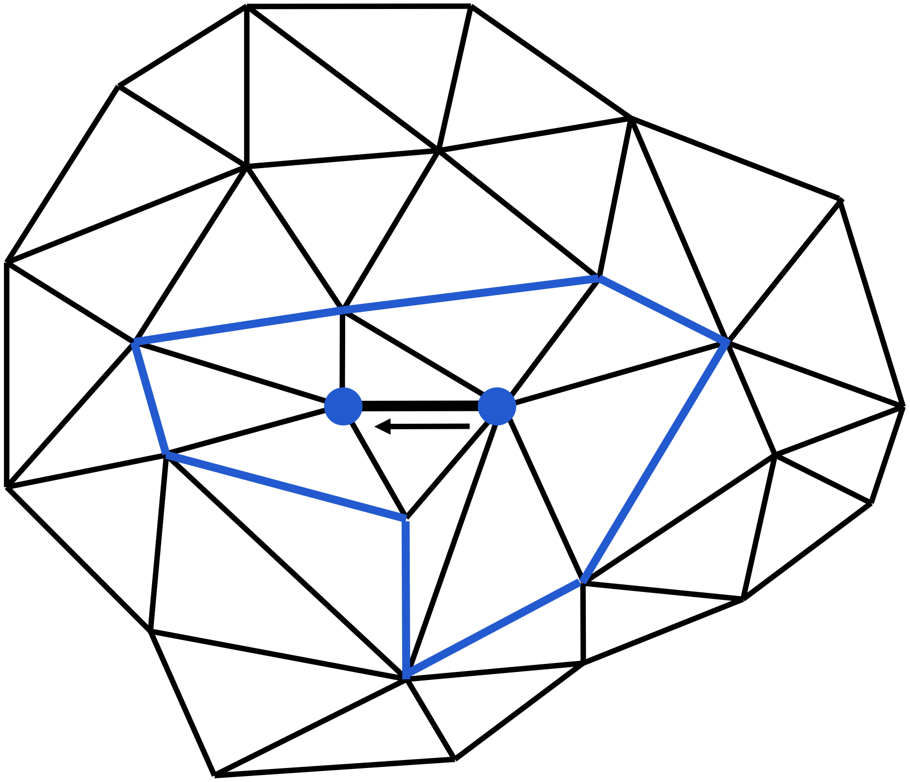
Edge Collapse



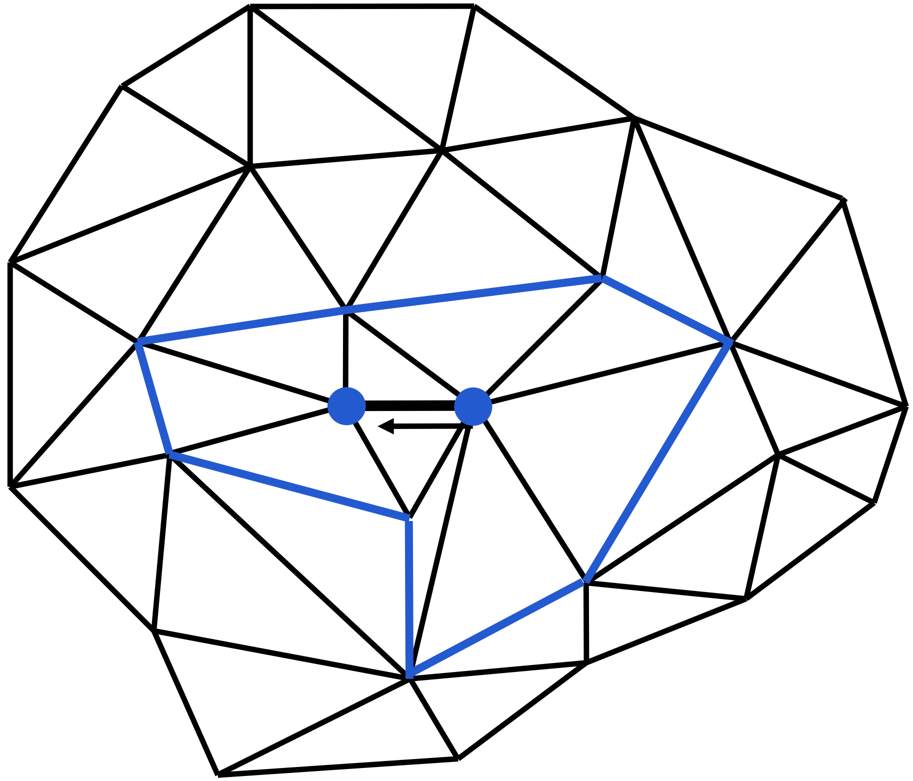
Edge Collapse



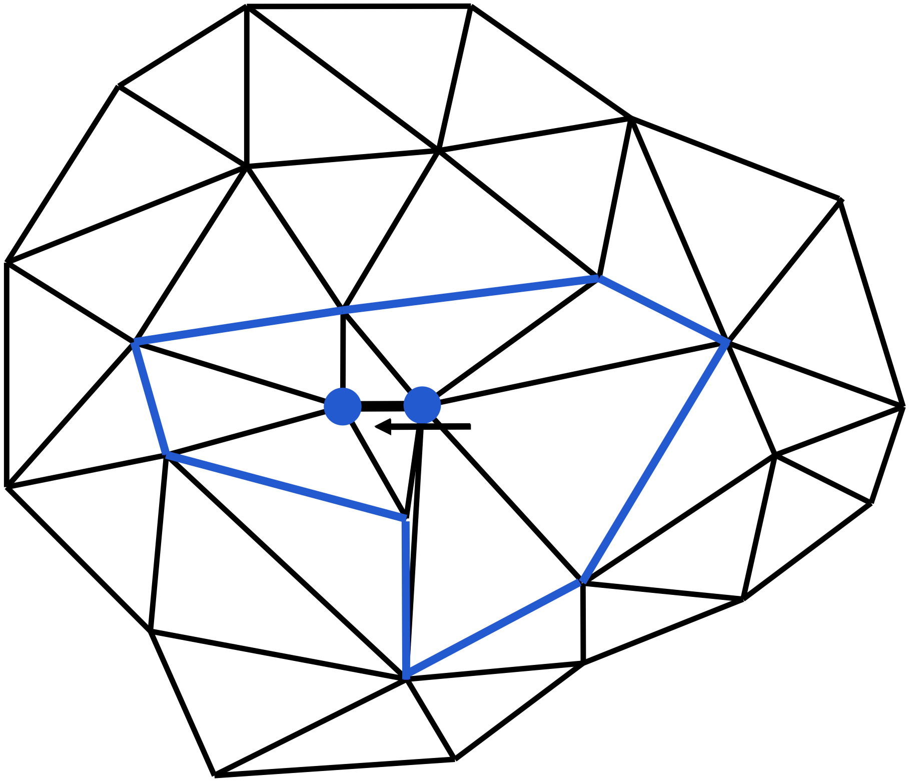
Edge Collapse



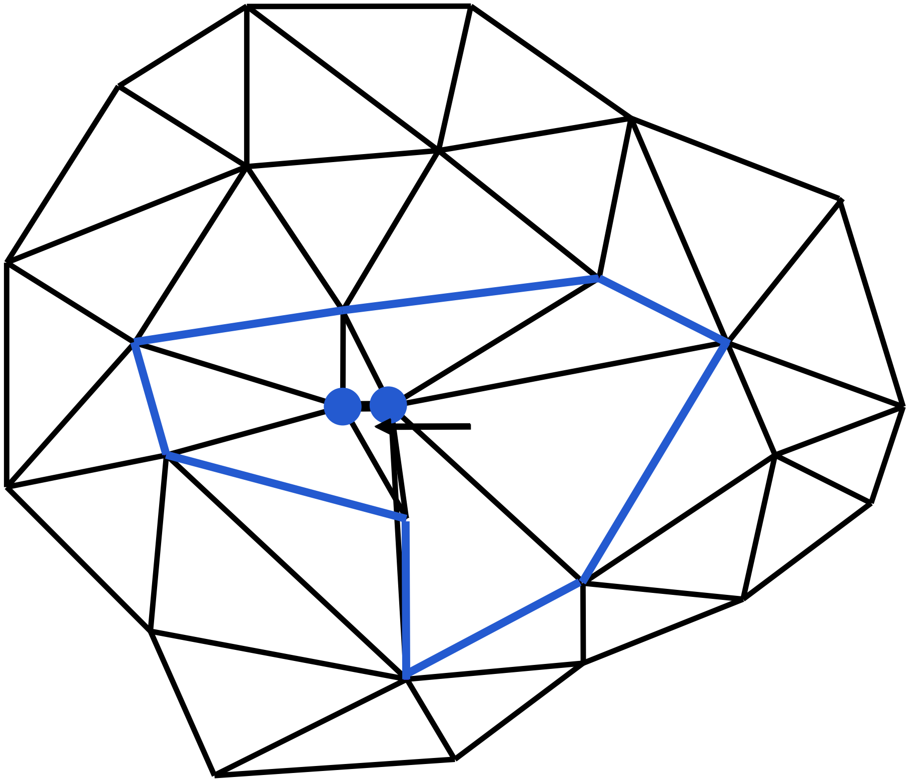
Edge Collapse



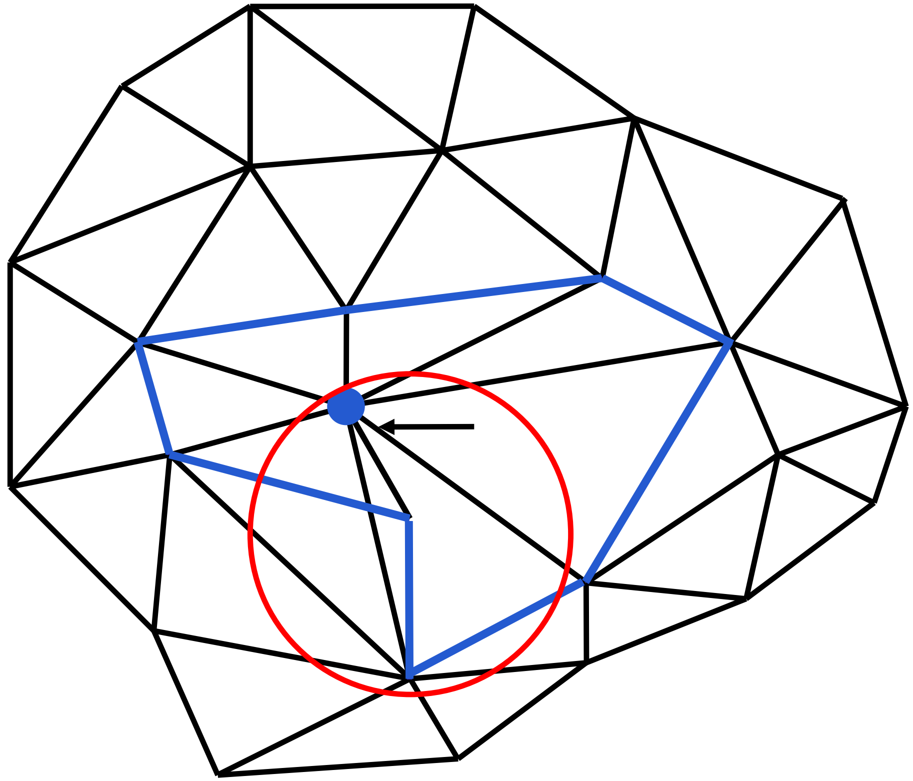
Edge Collapse



Edge Collapse



Edge Collapse

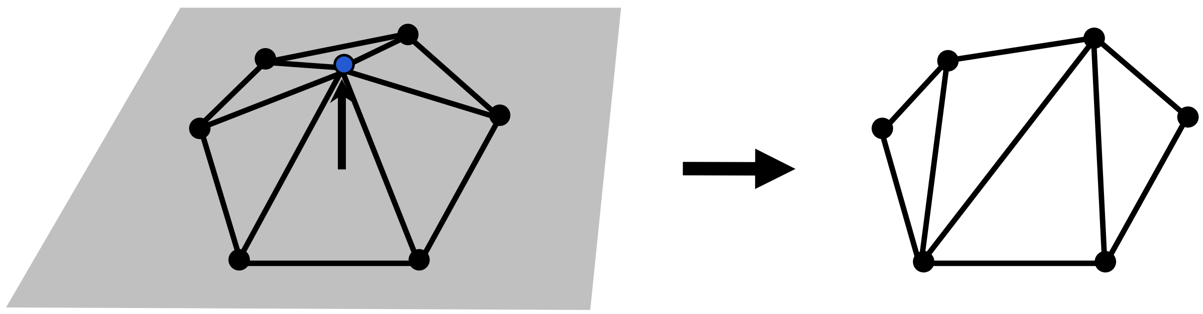


Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes

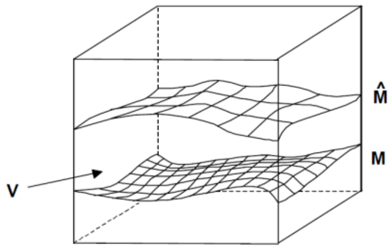
Local Error Metrics

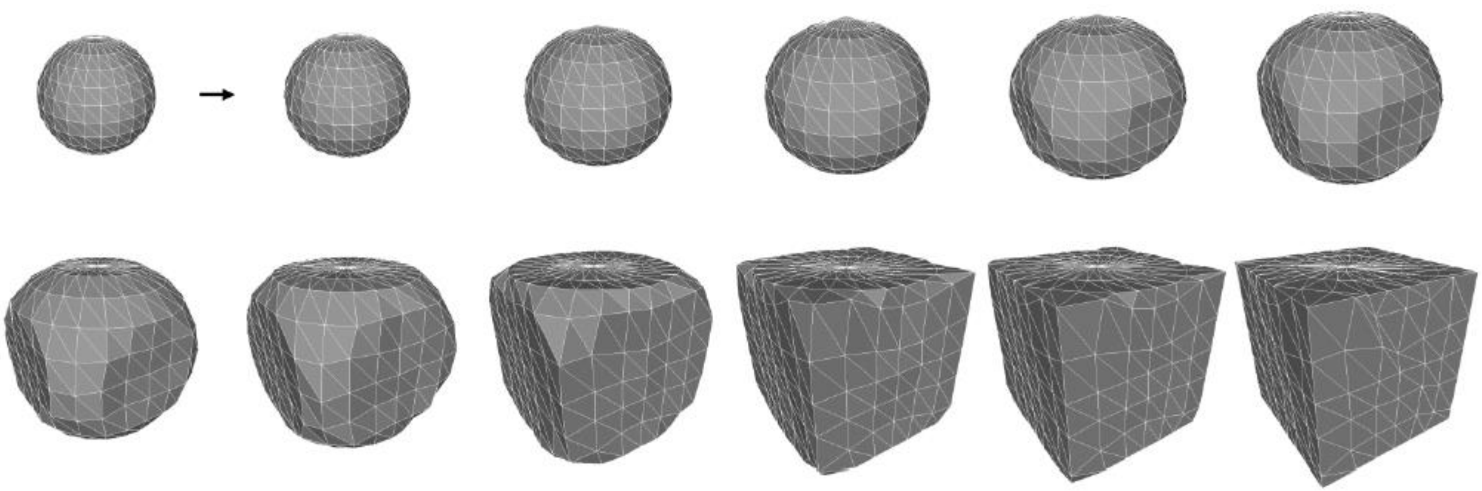
- Local distance to mesh [Schroeder et al. 92]
  - Compute average plane
  - No comparison to *original* geometry



Local Error Metrics

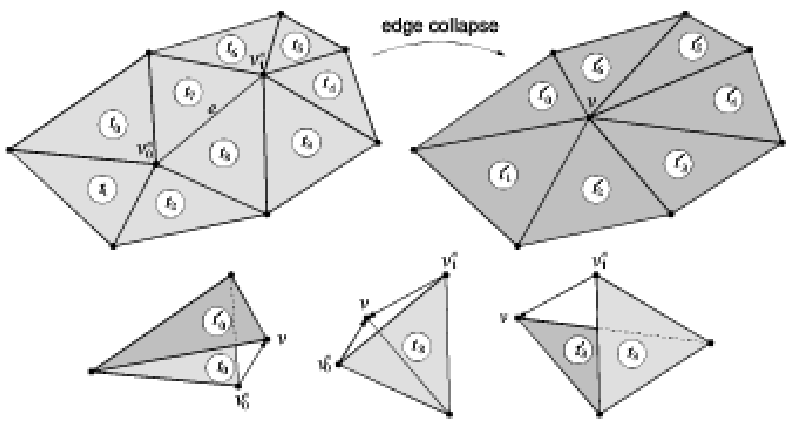
Volume [A. et al.]. *Mesh Approximation using a Volume-Based Metric*. Pacific Graphics 99.





Local Error Metrics

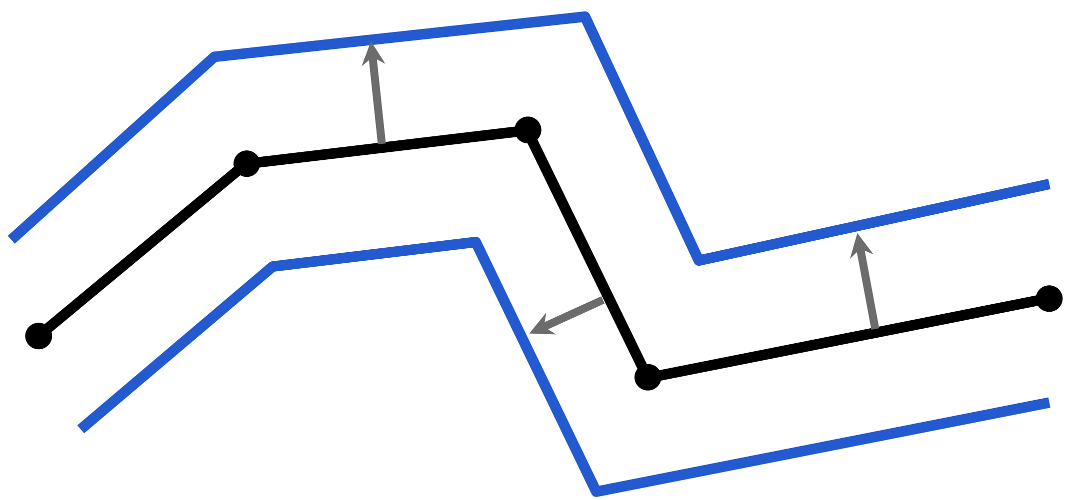
- Volume preserving [Lindstrom-Turk]. *Fast and memory efficient polygonal simplification*. *IEEE Visualization* 98.



Implemented in

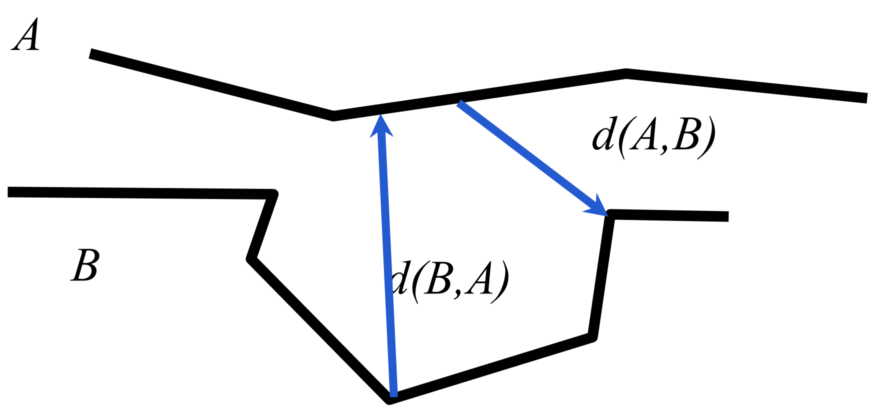
Global Error Metrics

- Simplification envelopes [Cohen et al. 96]
  - Compute (non-intersecting) offset surfaces
  - Simplification guarantees to stay within bounds



Global Error Metrics

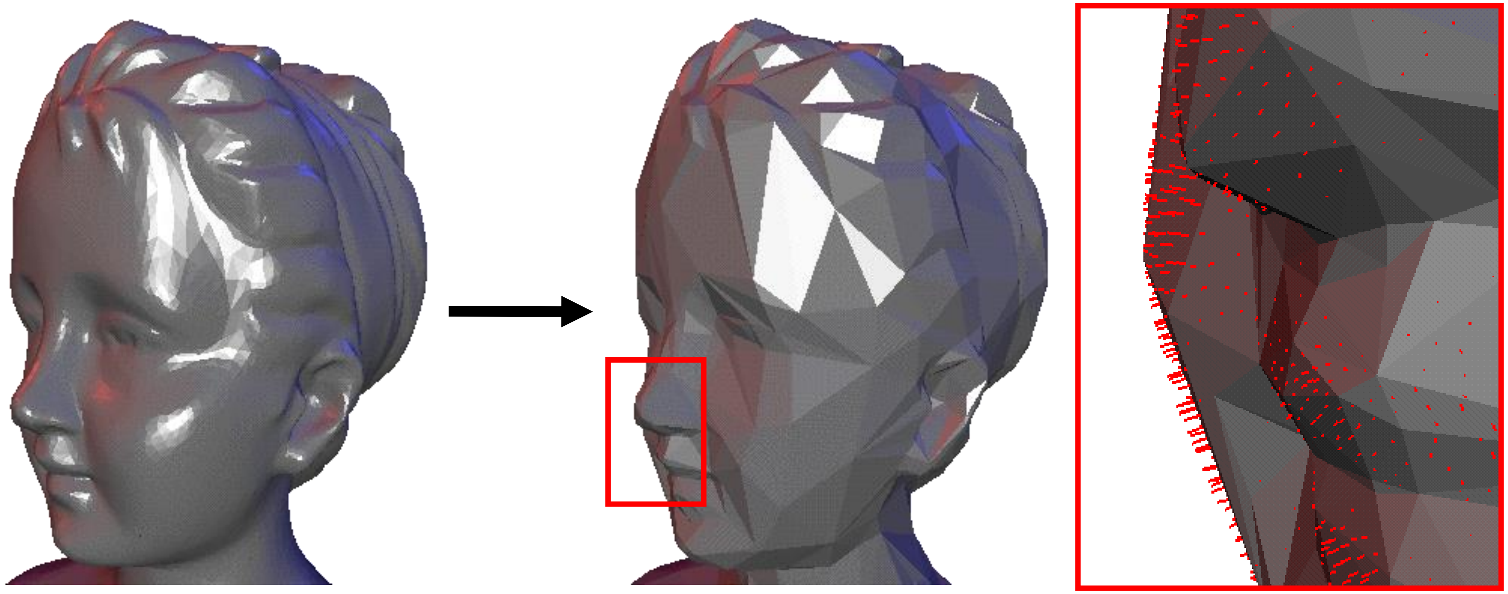
- (Two-sided) Hausdorff distance: Maximum distance between two shapes
  - In general $d(A,B) \neq d(B,A)$
  - Compute-intensive



Valette et al. *Mesh Simplification using a two-sided error minimization*. 2012.

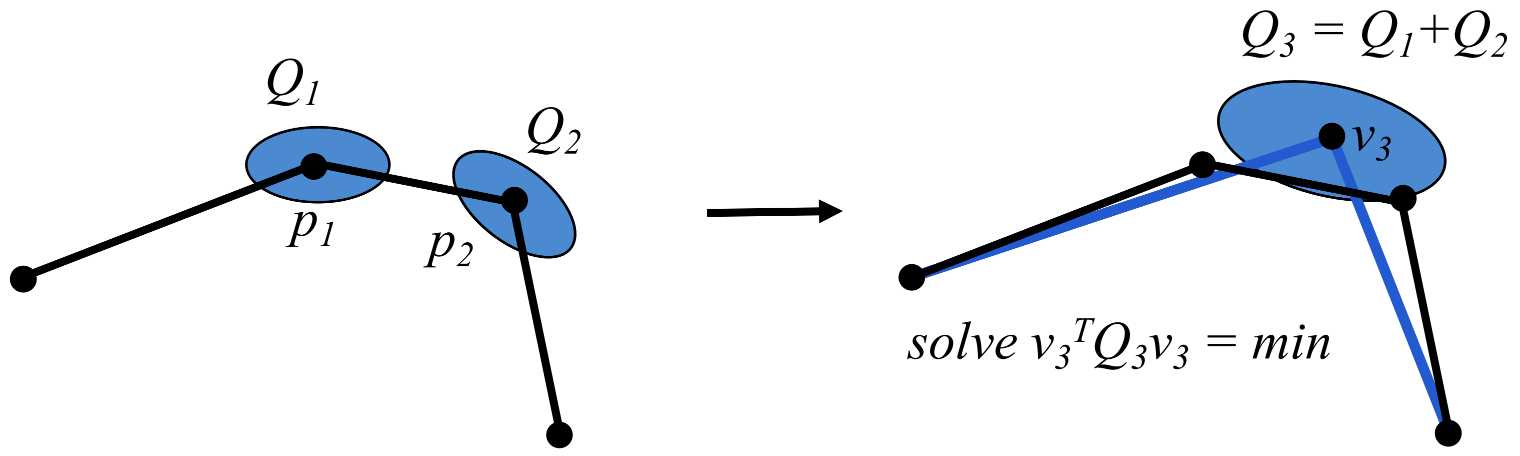
Global Error Metrics

- One-sided Hausdorff distance
  - From original vertices to current surface

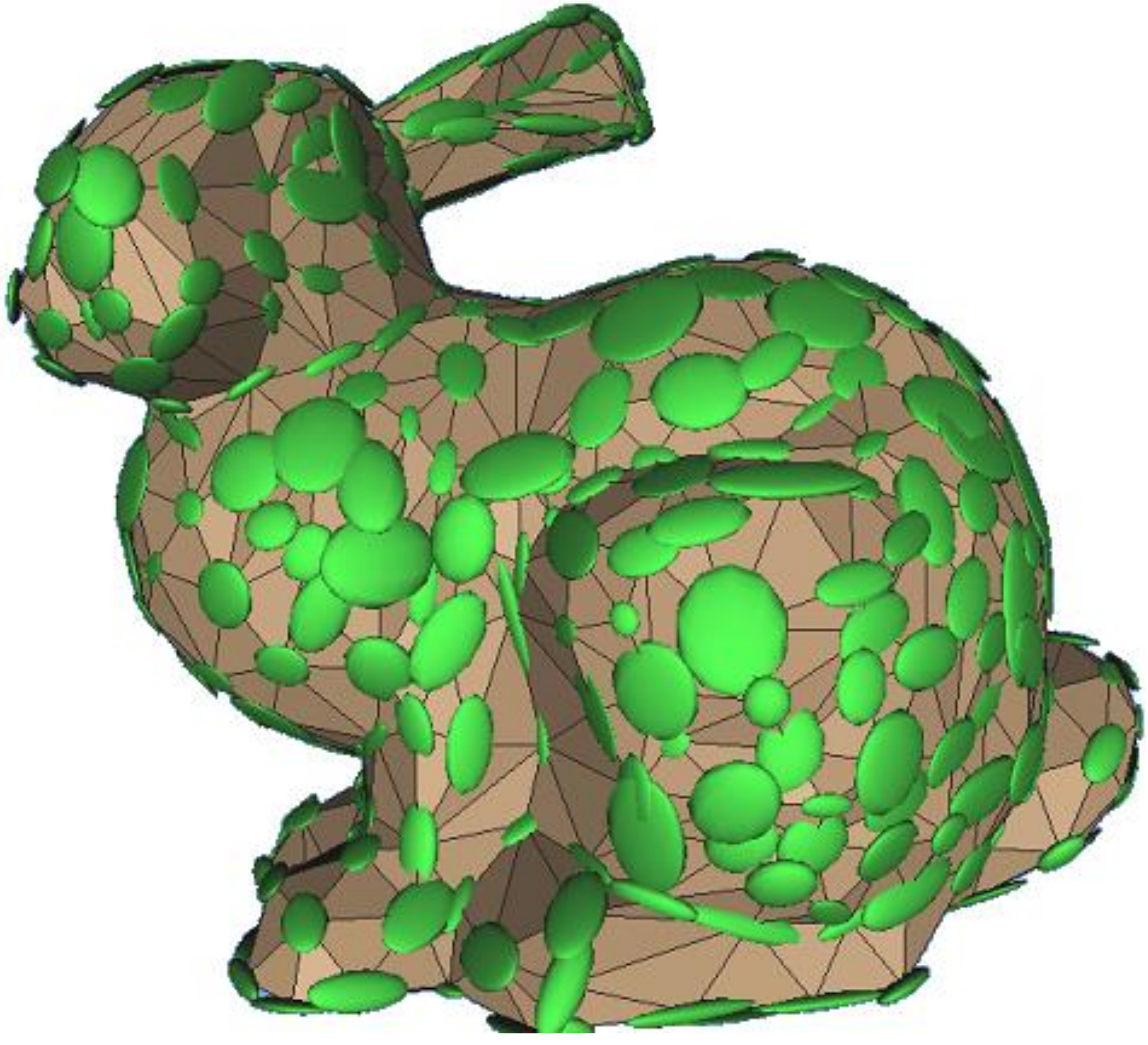


Global Error Metrics

- Error quadrics [Garland, Heckbert 97]
  - Squared distance to planes at vertex
  - No bound on true error



Error Quadrics

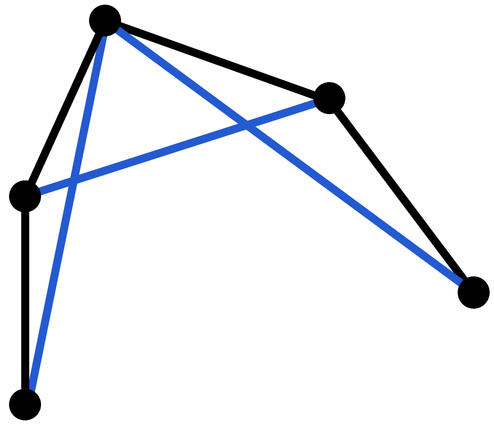


Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes

Fairness Criteria

- Rate quality of decimation operation
  - Approximation error
  - Triangle shape
  - Dihedral angles
  - Valence balance
  - Color differences
  - ...



Fairness Criteria

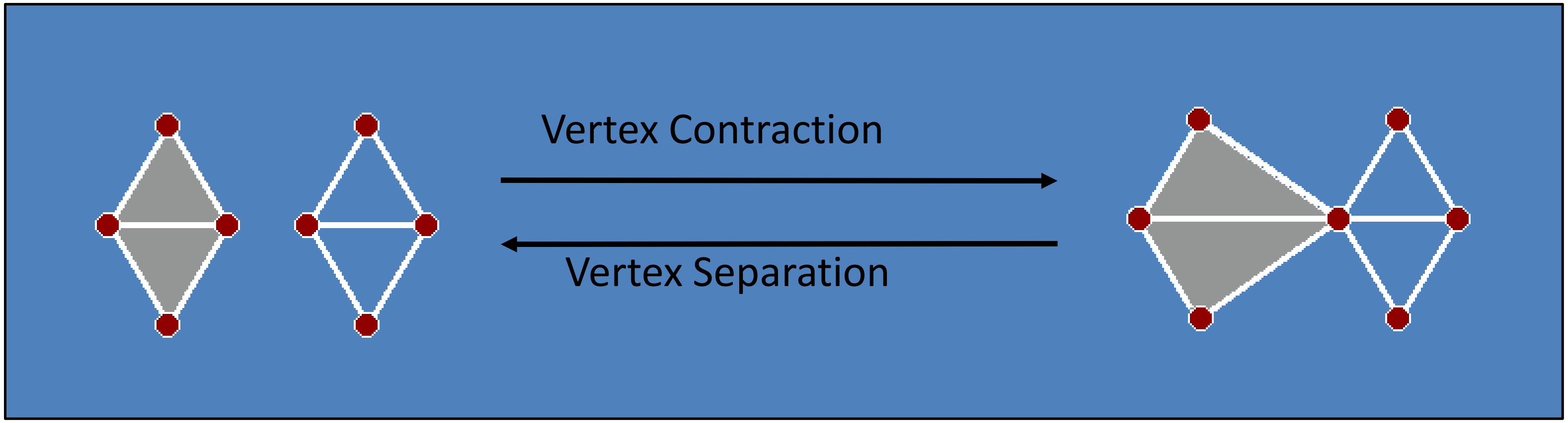
- Rate quality after decimation
  - Approximation error
  - Triangle shape
  - Dihedral angles
  - Valance balance
  - Color differences
  - ...

Incremental Decimation

- General Setup
- Decimation operators
- Error metrics
- Fairness criteria
- Topology changes

Topology Changes

- Merge vertices across non-edges
  - Changes mesh topology
  - Need *spatial neighborhood* information
  - Generates *non-manifold* meshes



Topology Changes

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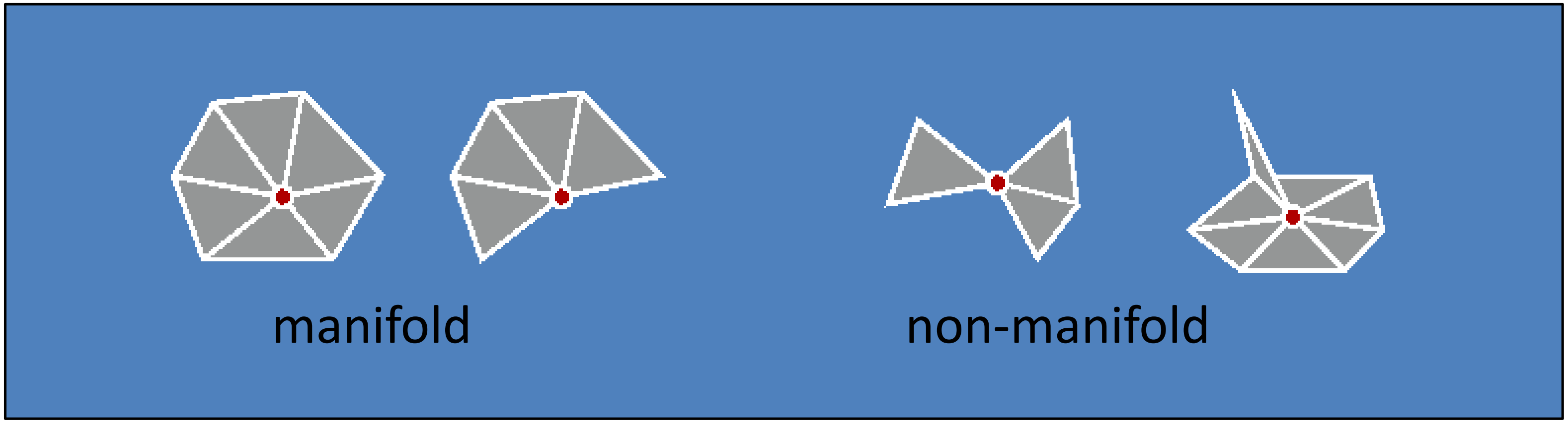


Comparison

- Vertex clustering
  - fast, but difficult to control simplified mesh
  - topology changes, non-manifold meshes
  - global error bound, but often not close to optimum
- Iterative decimation with quadric error metrics
  - good trade-off between mesh quality and speed
  - explicit control over mesh topology

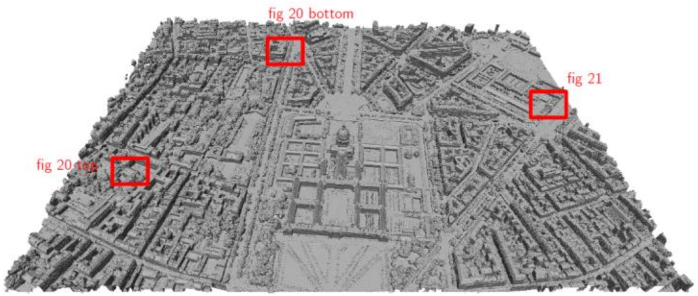
Simplification

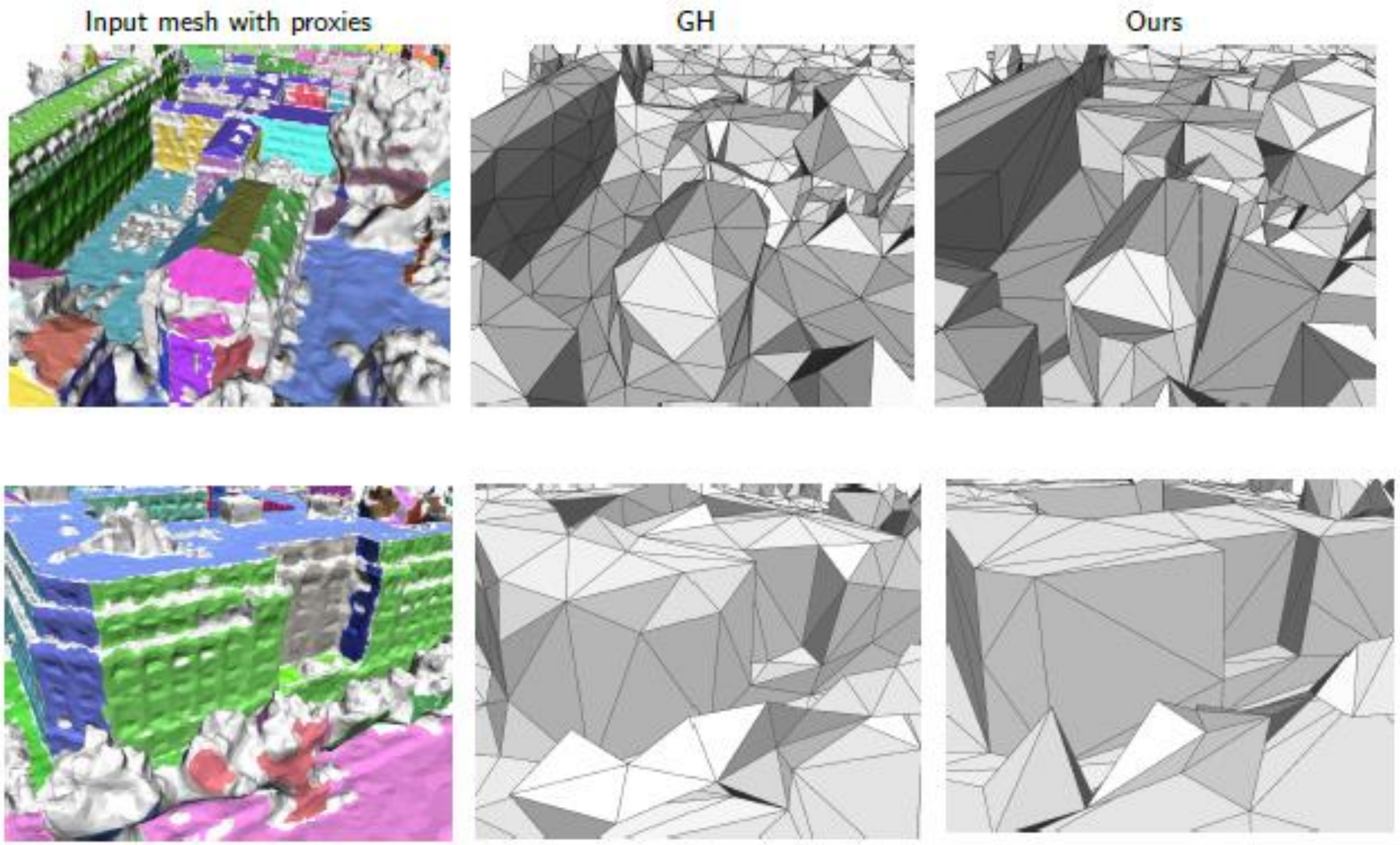
- Vertex Clustering
- Iterative Decimation
- **Extensions**

Extensions

- Out-of-core [Garland-Shaffer, Wu-Kobbelt, Isenburg]
- View-dependent [Hoppe]
- Structure-aware [Salinas, Lafarge, A.]

Structure-Aware





Salinas, Lafarge, A. Structure-Aware Mesh Decimation.
Computer Graphics Forum 2015

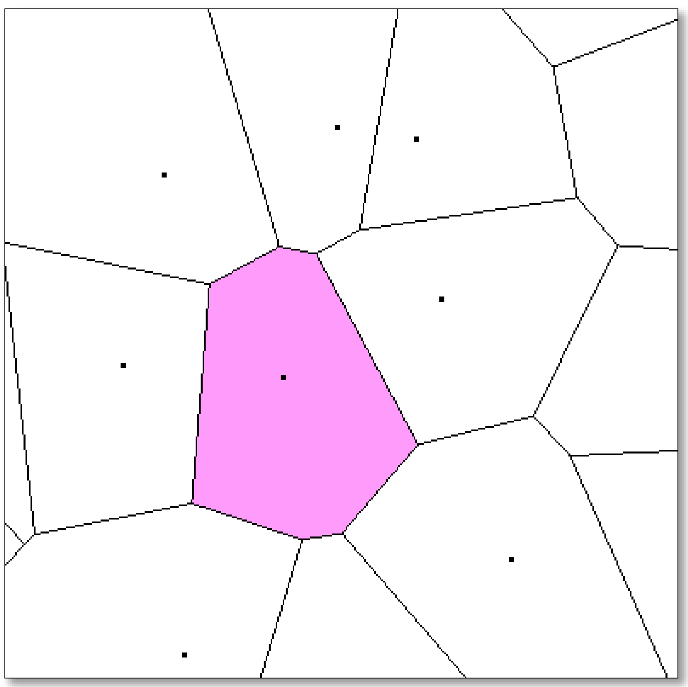
Approximation

Variational Shape Approximation

- Rationale: cast surface approximation as a variational k-partitioning problem

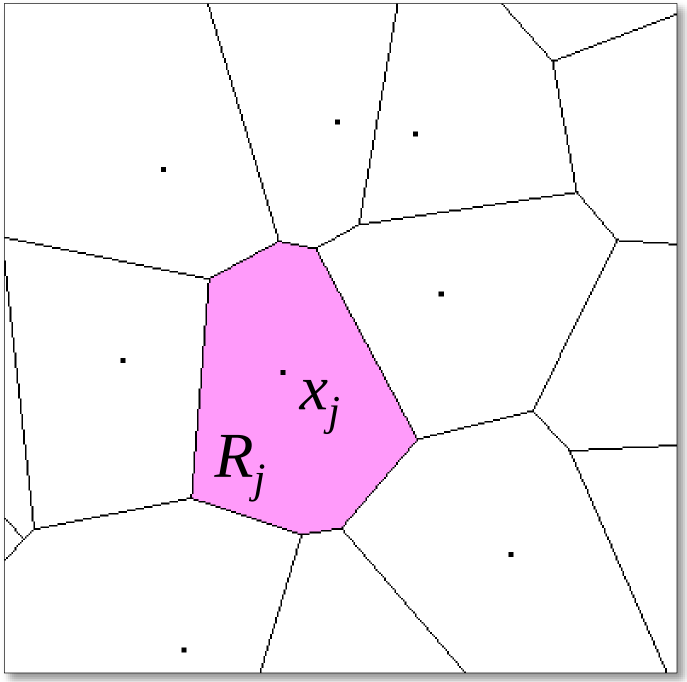
Cohen-Steiner, A., Desbrun. *Variational Shape Approximation*. SIGGRAPH 2004.

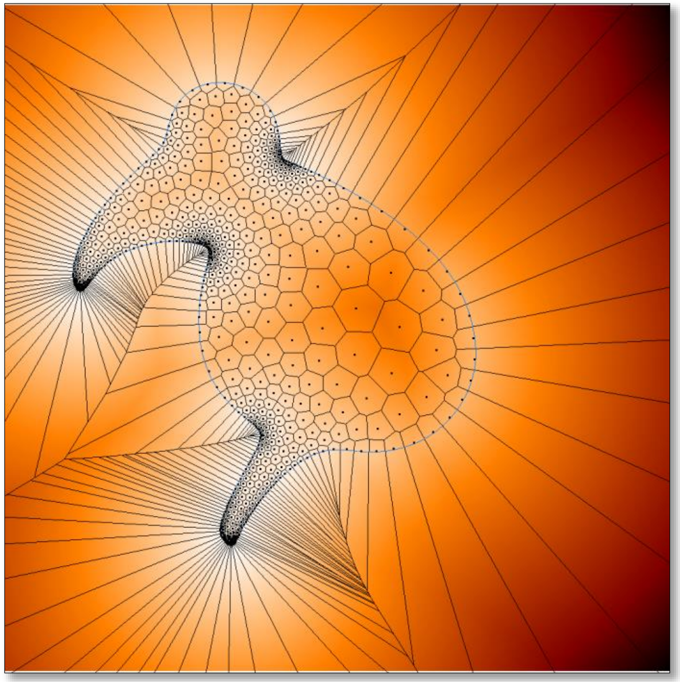
Simpler Setting: 2D Partitioning



Energy

$$E = \sum_{j=1..k} \int_{x \in R_j} \rho(x) \| x - x_j \|^2 \, dx$$

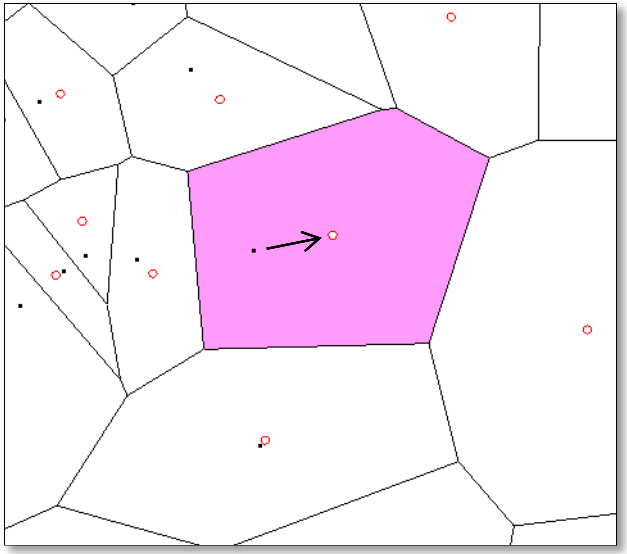




density function

Lloyd Iteration

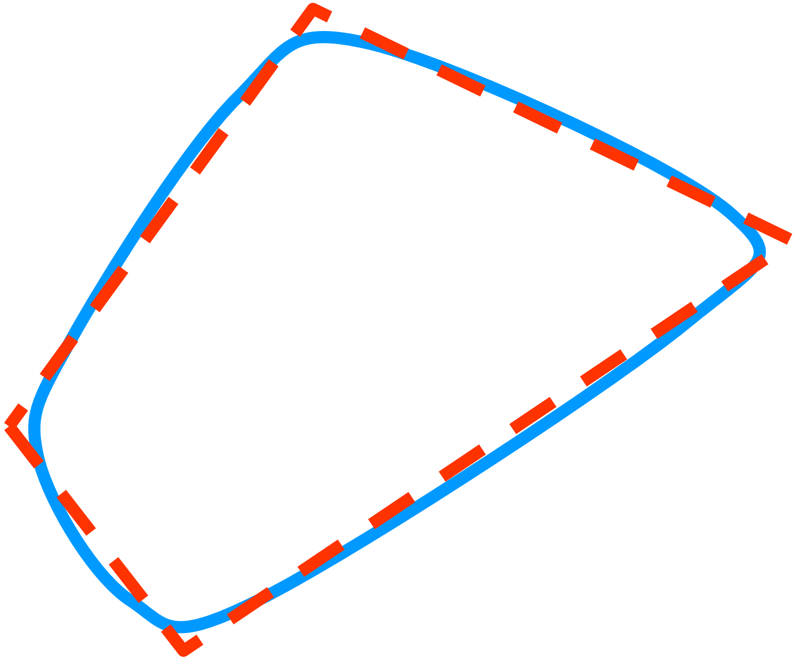
- Alternate:
  - Voronoi partitioning
  - Relocate sites to centroids
- Minimizes energy
  - Necessary condition for optimality: Centroidal Voronoi tessellation



demo

Variational Shape Approximation

- Rationale: cast surface approximation as a variational k-partitioning problem
  - for each region, find best-fit *linear proxy*
    - "best fit" for a given metric



demo

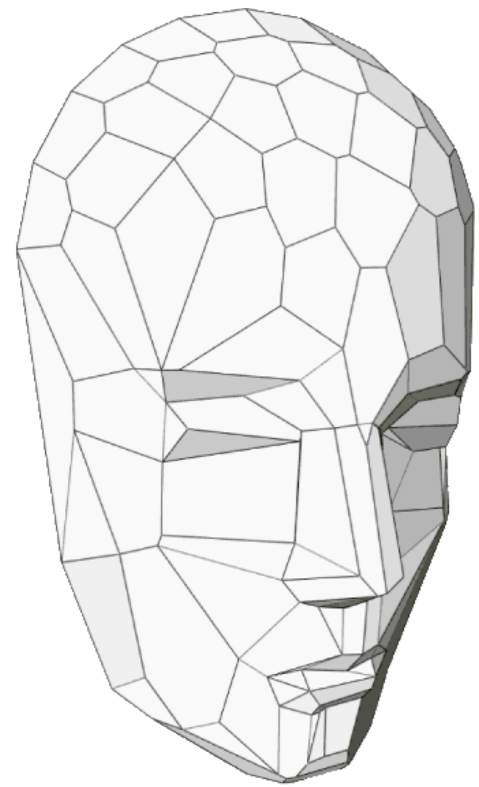
Variational Shape Approximation

- Distortion
  = integrated error between region and proxy
- Total distortion = sum of proxy distortion
- Best k-approximation = minimum distortion

Overview

initial mesh
+ partition

associated
proxies



proxy-based
remeshing

K-Means Clustering

Starting with k-generators

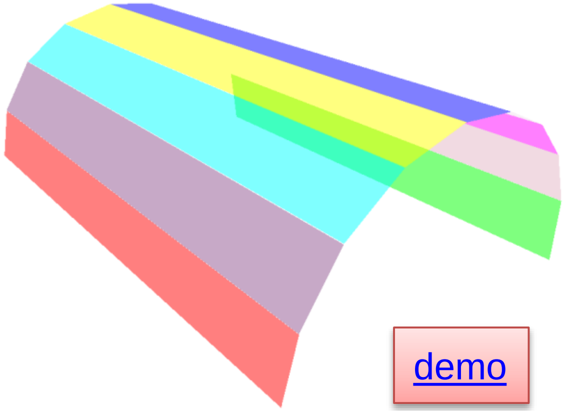
Alternate:

- cluster by closest proximity (creates regions R_j)
- find new generators c_j of regions R_j

Partition Optimization

Clustering for Approximation

- Replace points by proxies
- Min approximation error
- Equi-distribute energy among proxies

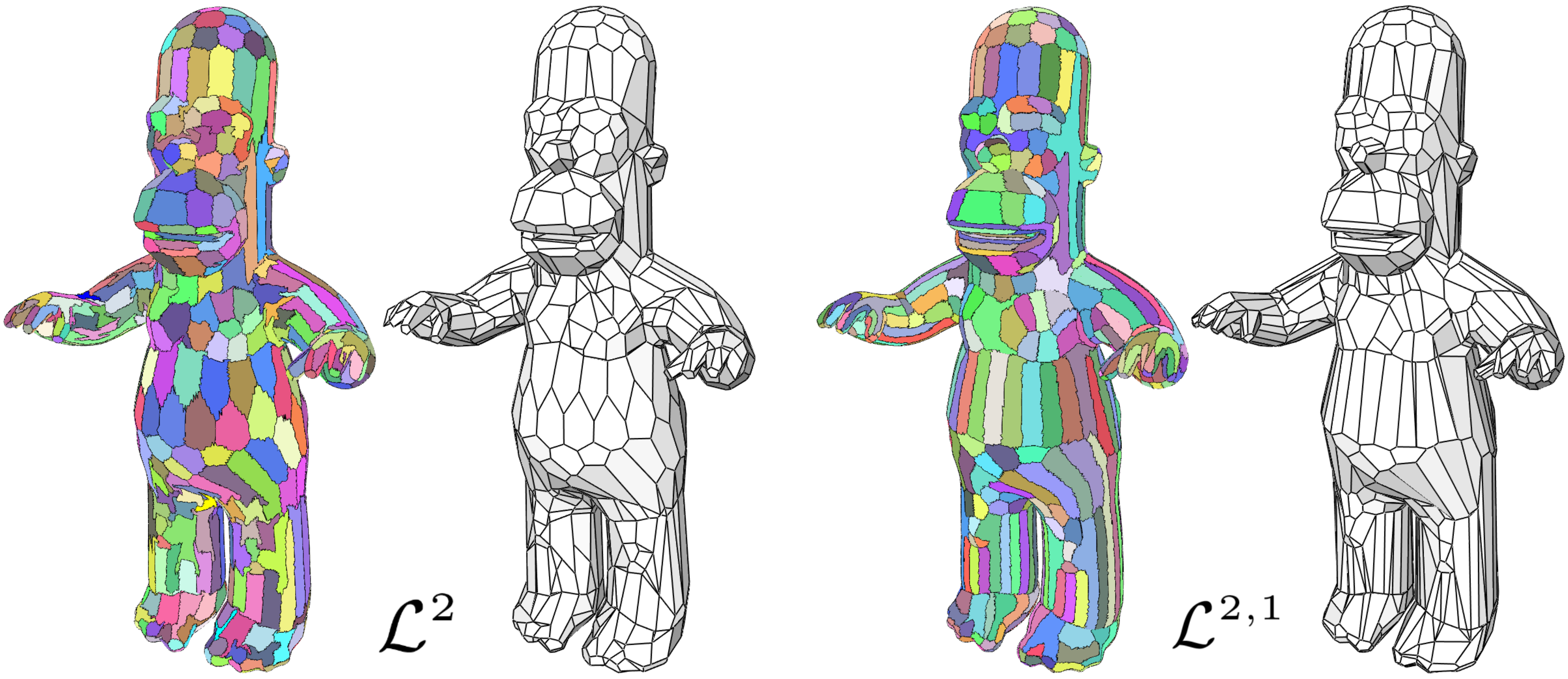


demo

Error Metrics

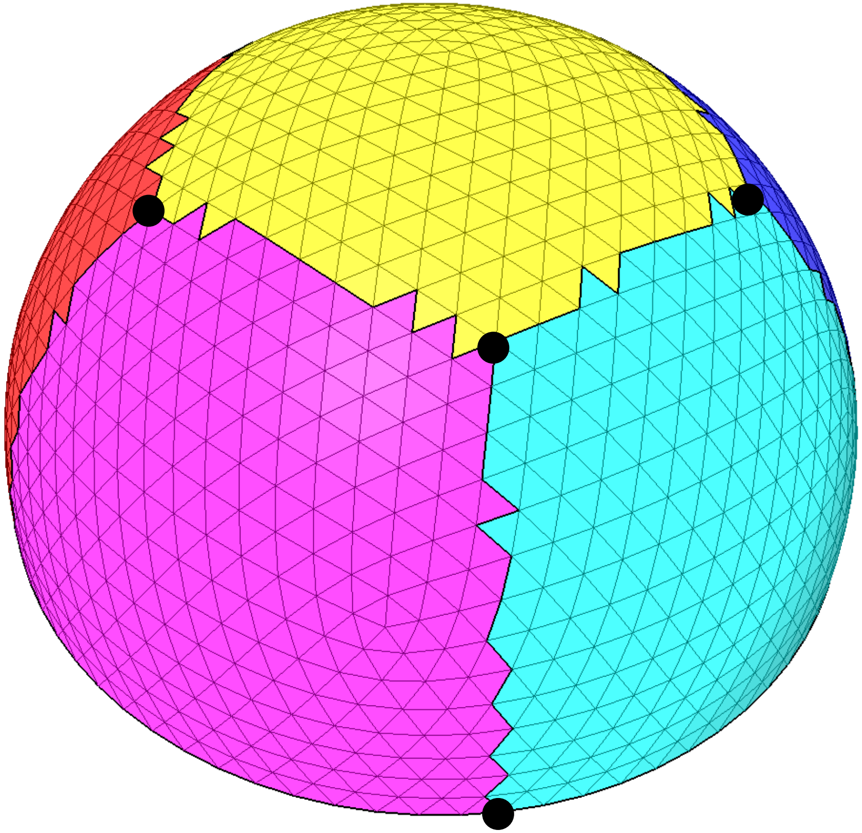
- L^2
  - asymptotically, aspect ratio is $\sqrt{\kappa_1/\kappa_2}$
  - hyperbolic regions troublesome
    - no unique minimum
  - convergence in L^2 does not guarantee in normals
    - example: Schwarz's Chinese lantern
    - [Shewchuck 04] gradient bounds harder than interpolation
- $L^{2,1}$ $\iint_{x \in X} \|\mathbf{n}(x) - \mathbf{n}_i\|^2 dx$
  - asymptotically, aspect ratio is κ_1/κ_2
  - hyperbolic regions ok
  - captures normal field

L^2 vs. $L^{2,1}$



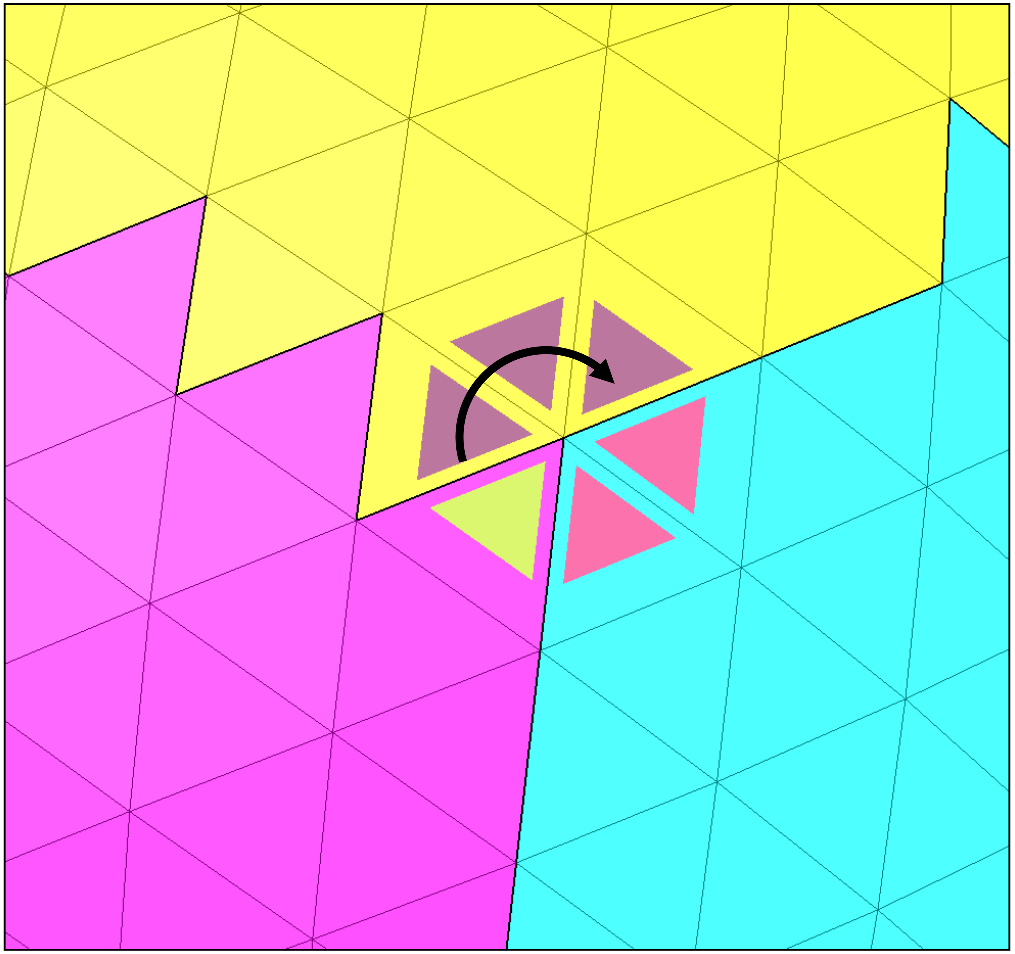
Triangulation

- node vertex
  - where 3+ regions meet
  - 2+ on boundary



Triangulation

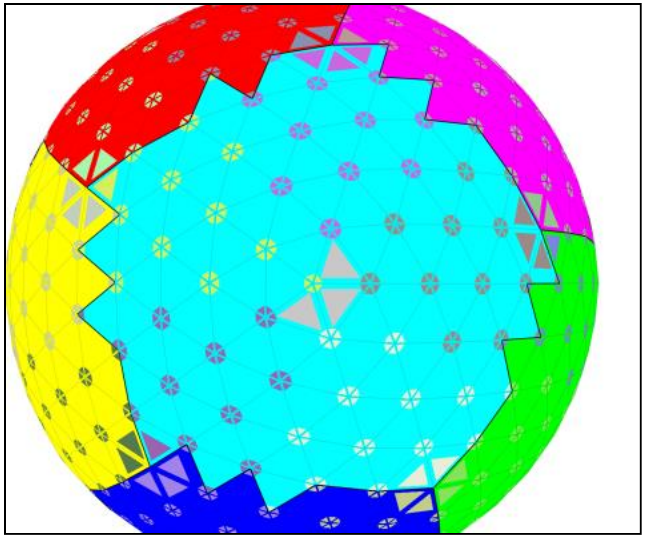
- node wedge



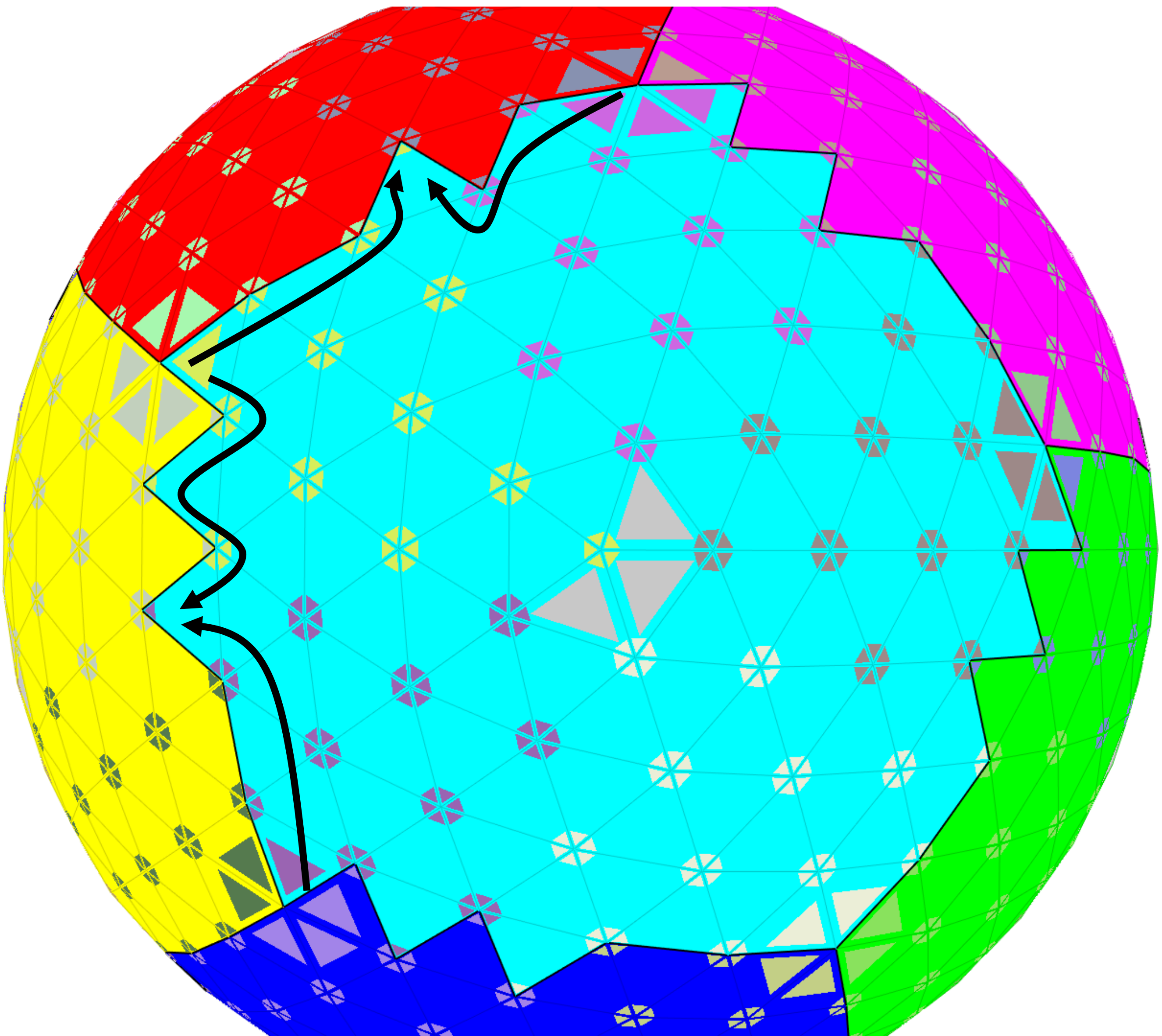
Triangulation

- Two-pass flooding algorithm (~multi-source Djisktra's shortest path algorithm)

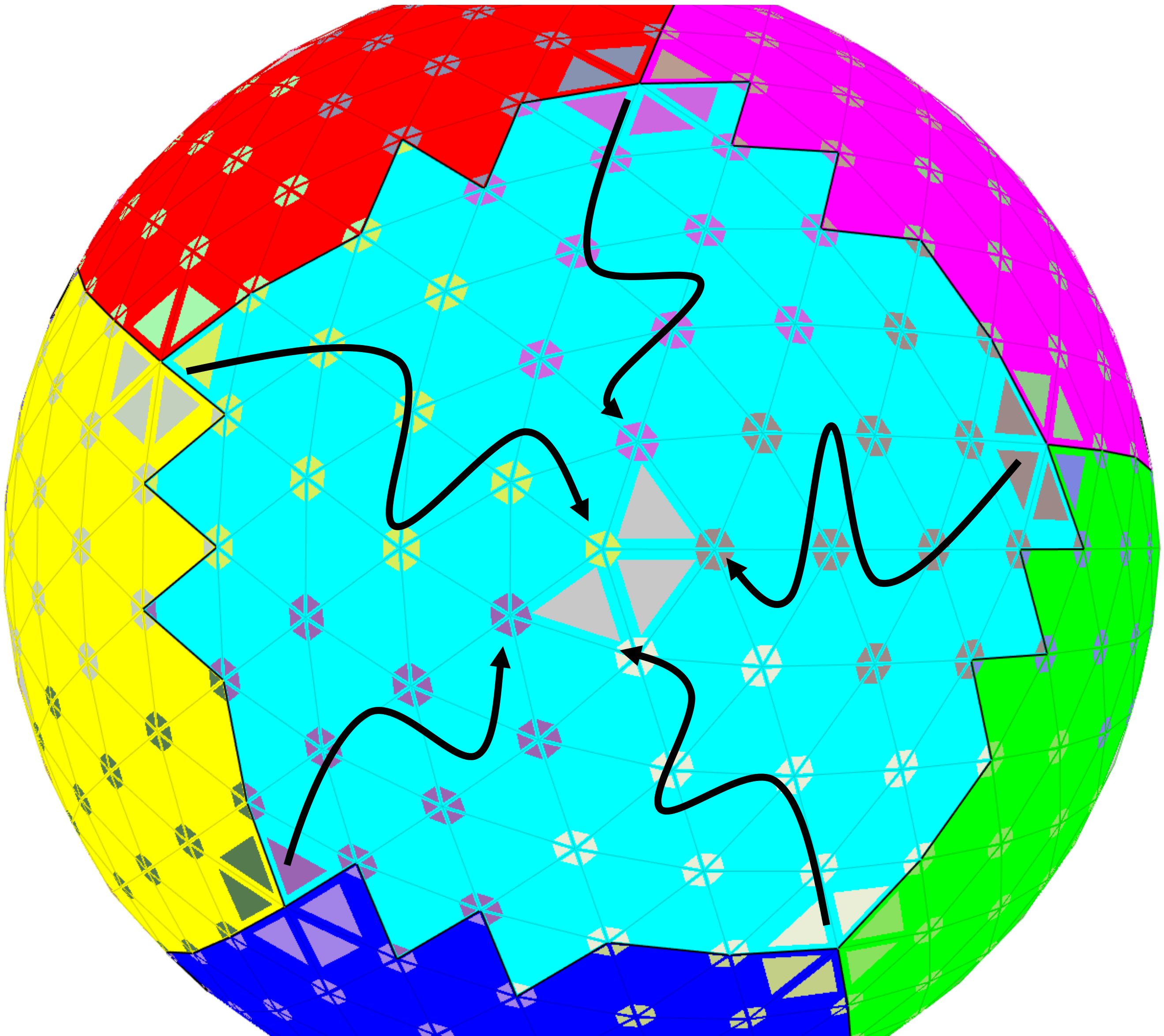
- first pass: flood only region boundaries (to enforce the *constrained edges)*
- second pass: flood interior areas



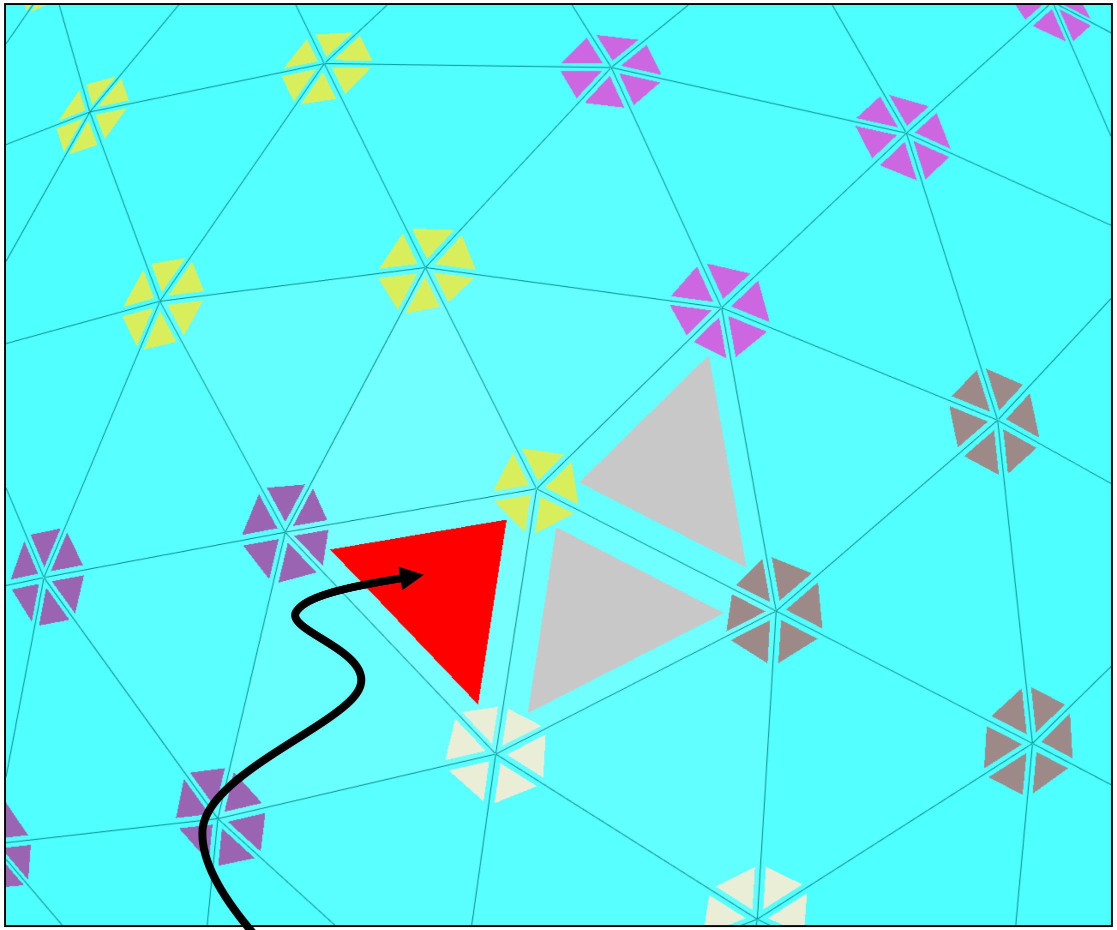
First pass



Second pass

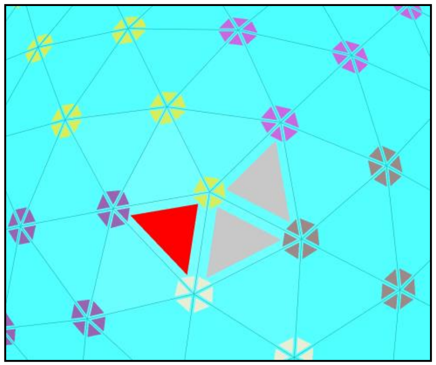


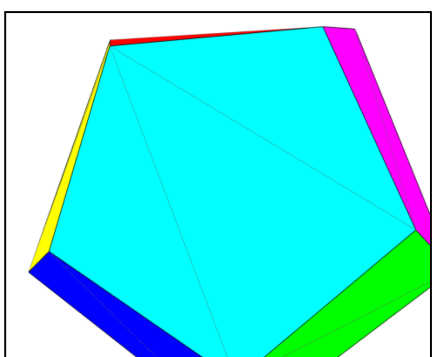
Triangulation

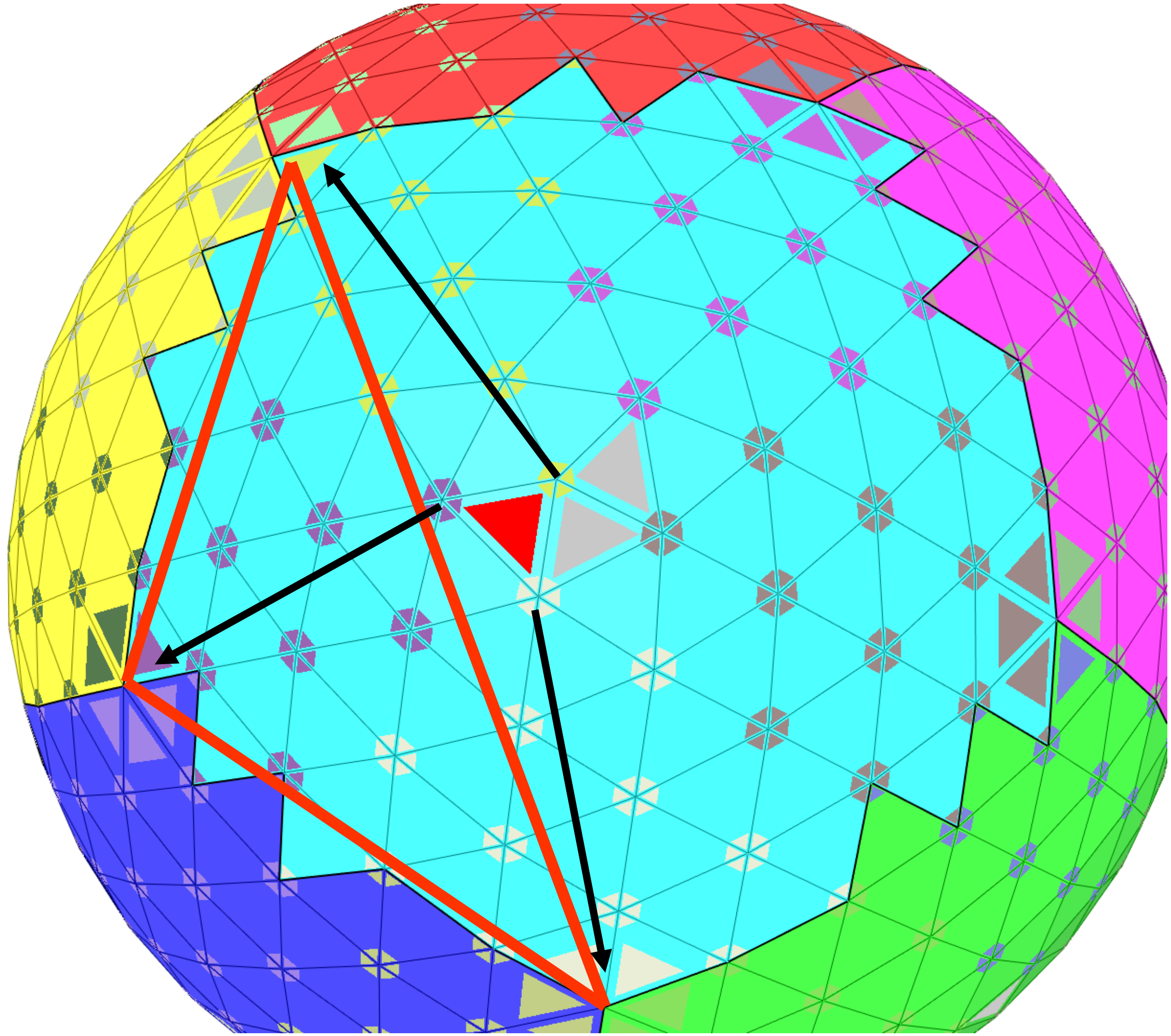


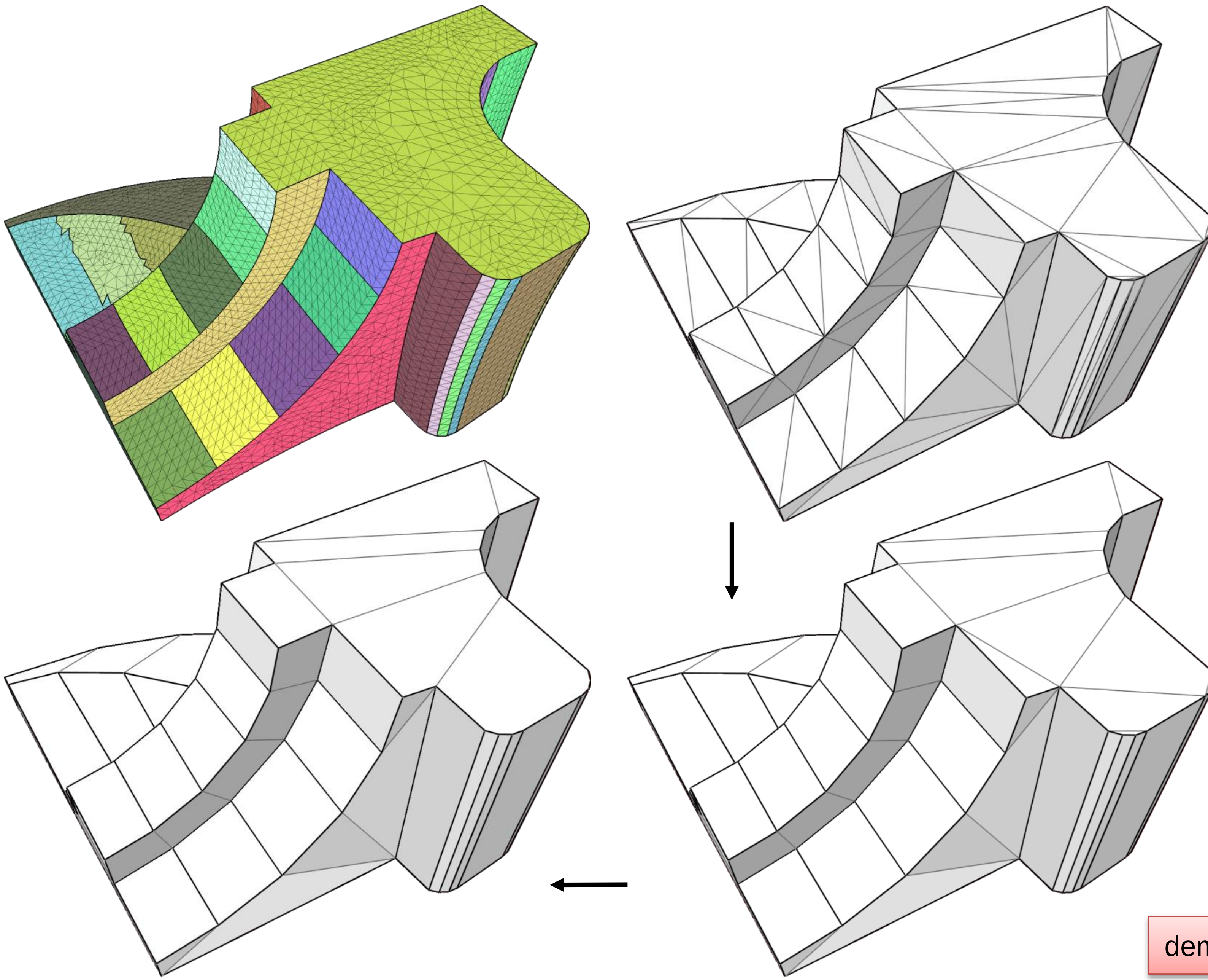
connect 3 source wedges

Triangulation



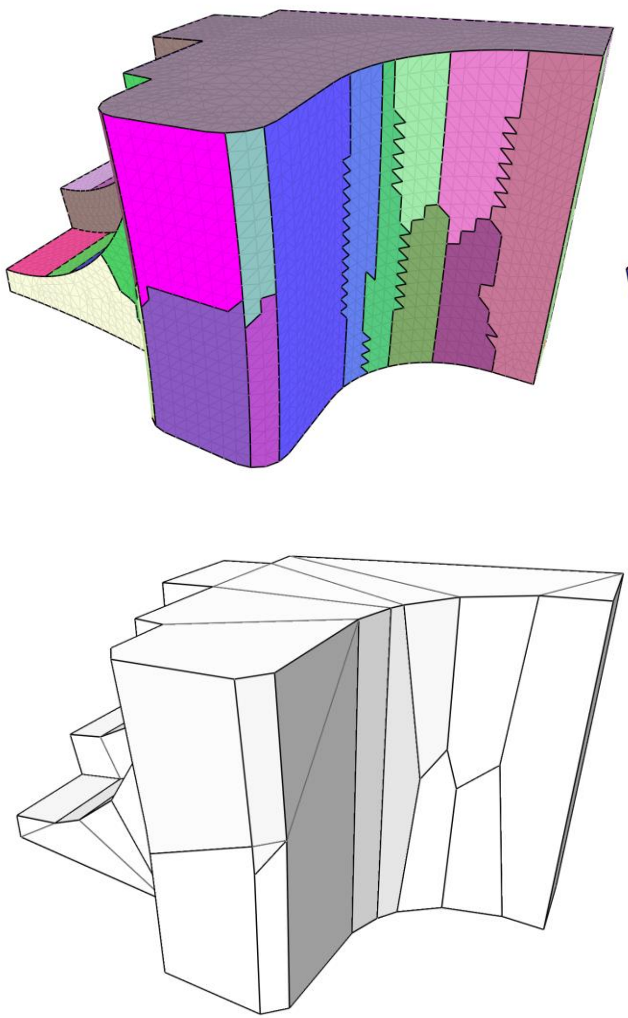




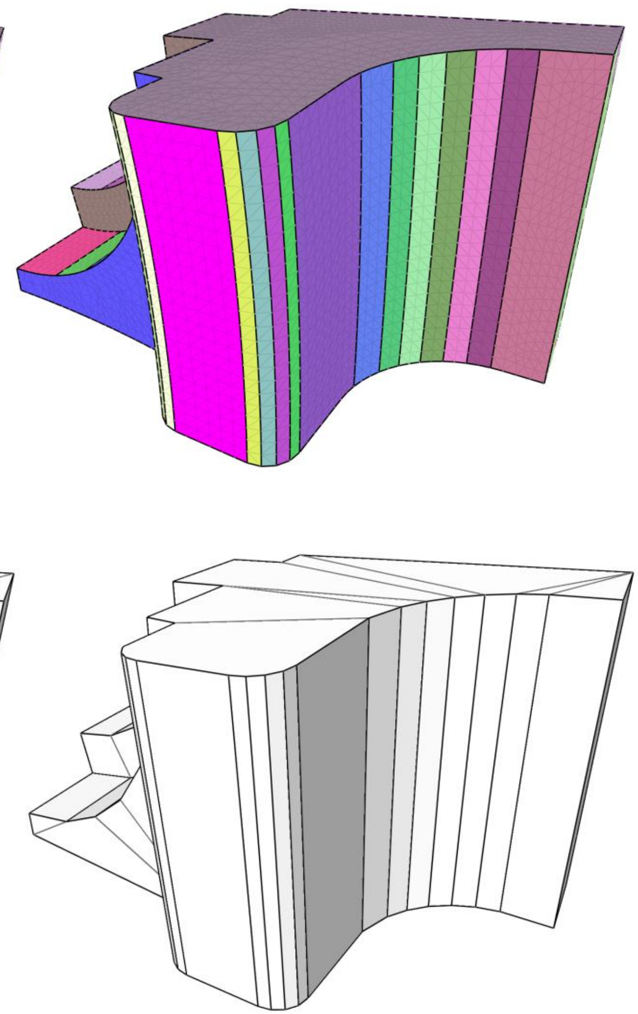


demo

Metrics

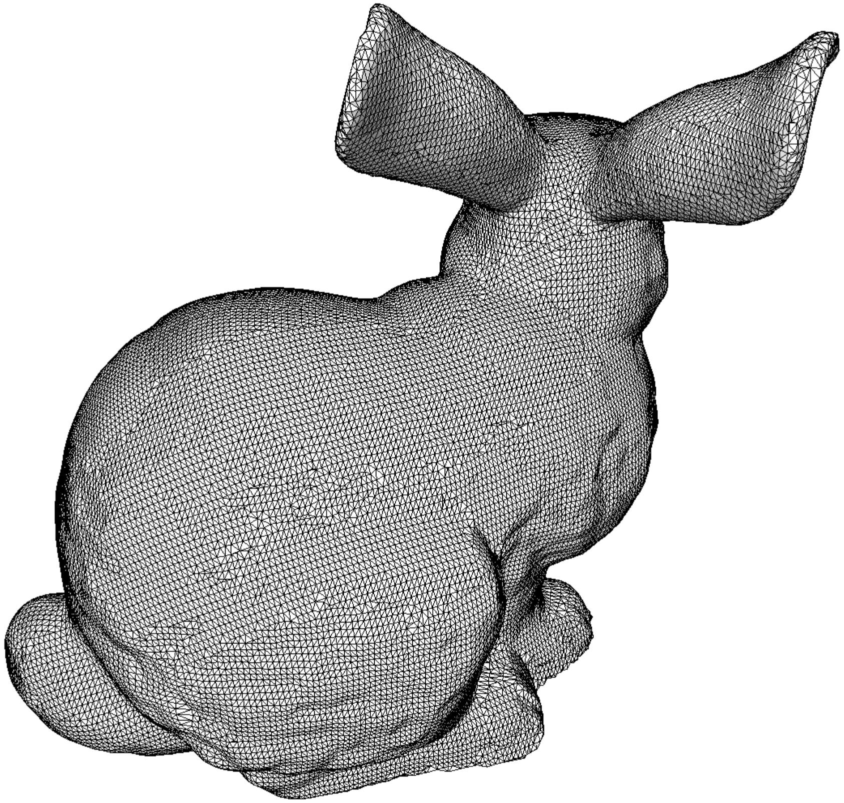


L^2

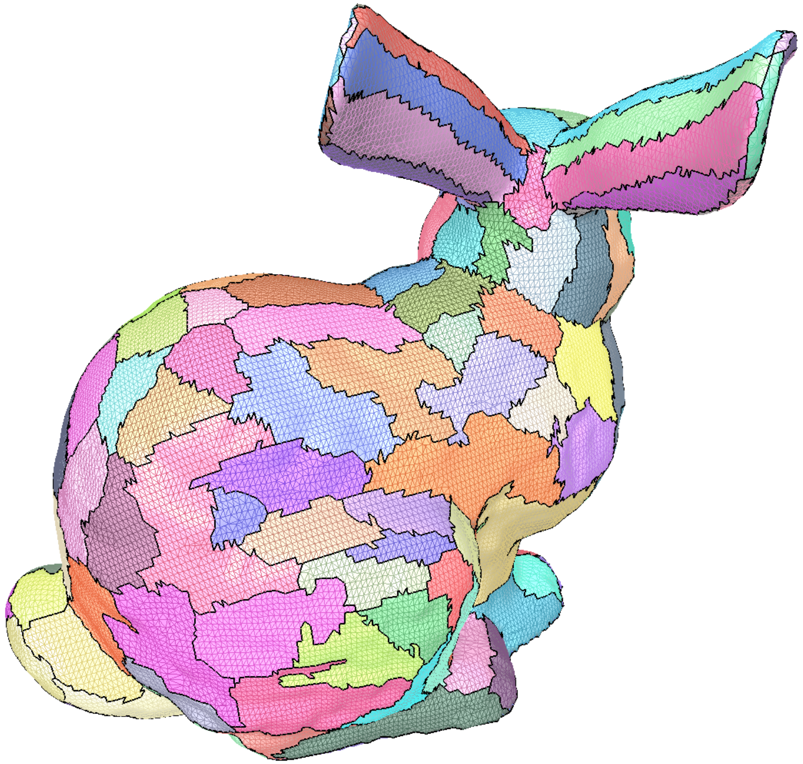


$L^{2,1}$

Example

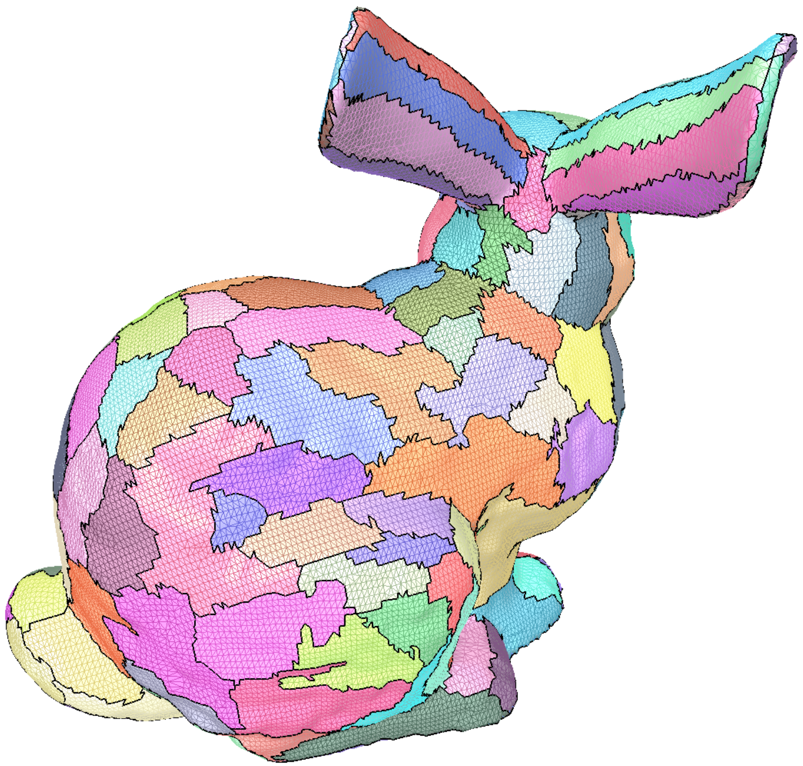


input

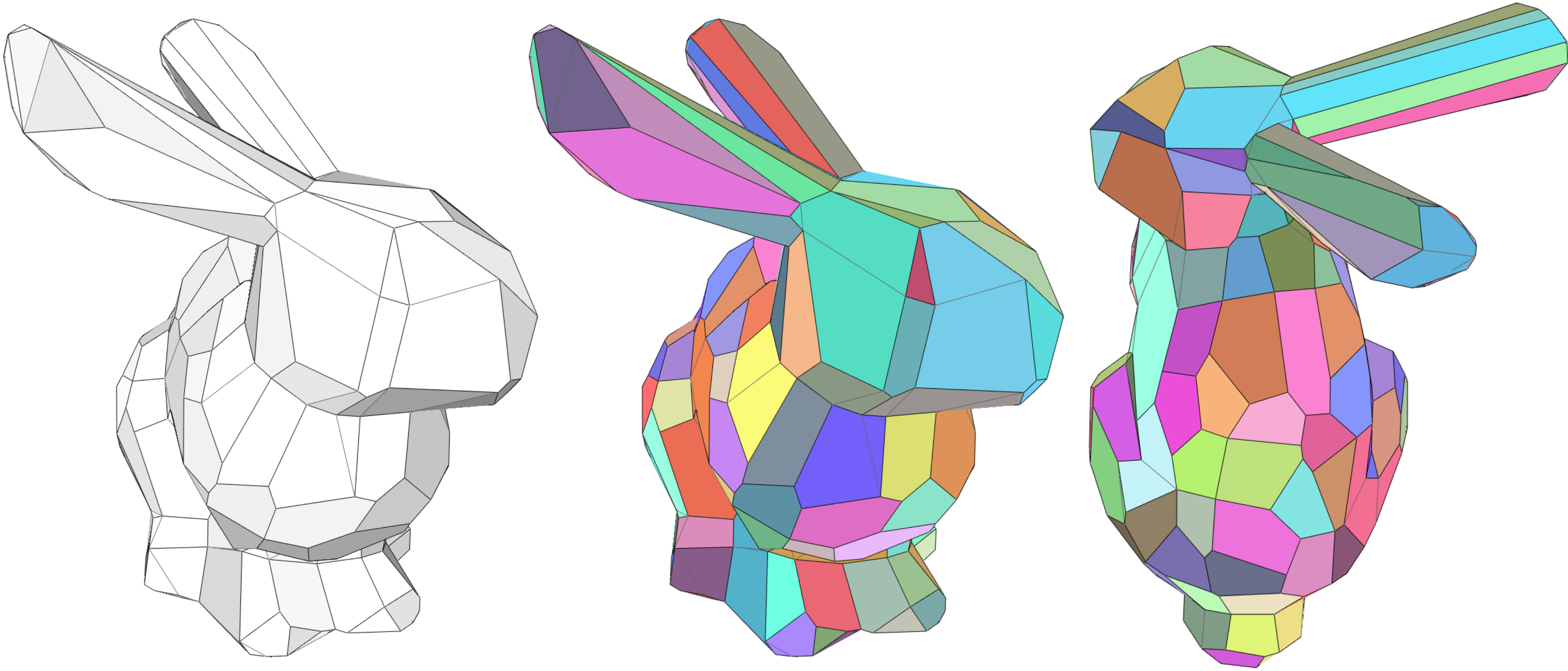


200 proxies

Example



Example



400KT


5KP

400KT 5KP

Guarantees

Guarantees?

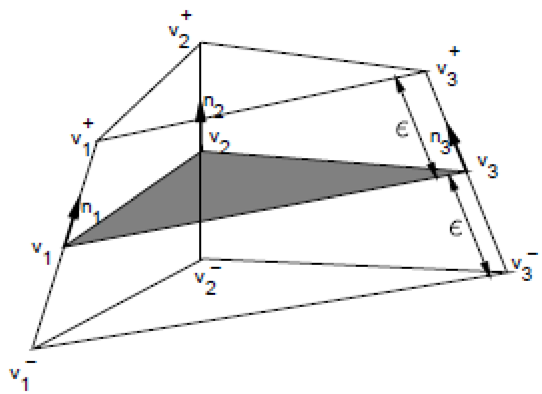
- **Error bound**
  - One-sided Hausdorff [Klein, Botsch]
  - Two-sided Hausdorff [Valette]
- **Topology**
  - Intersection-free
  - Homeomorphism [Oudot, Boissonnat]
  - Isotopy [Chazal, Cohen-Steiner]

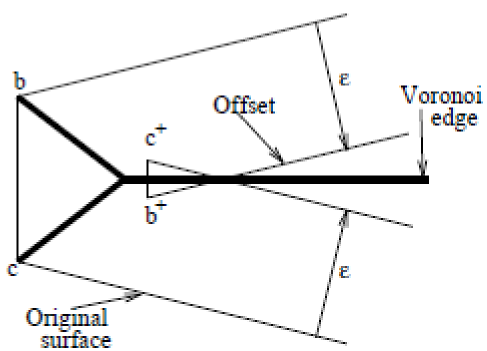
Tolerance Volume

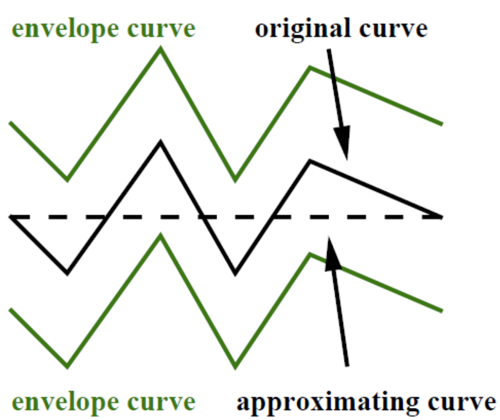
- **Simplification Envelopes**.
  Cohen et al. SIGGRAPH 96.
- **Multiresolution Decimation based on Global Error**.
  Ciampalini et al. The Visual Computer 97.
- **Surface Simplification Inside a Tolerance Volume**.
  Guéziec. IBM RR 97.
- **Adaptively Sampled Distance Fields**.
  Frisken et al. SIGGRAPH 2000.
- **Permission Grids: Practical, Error-Bounded Simplification**.
  Zelinka & Garland. ACM TOG 2002.
- **GPU-based Tolerance Volumes for Mesh Processing**.
  Botsch et al. Pacific Graphics 2004.
- **Simplification of Surface Mesh using Hausdorff Envelope**.
  Borouchaki & Frey. 2005.

Tolerance Volume

- **Simplification Envelopes** [Cohen et al 96]



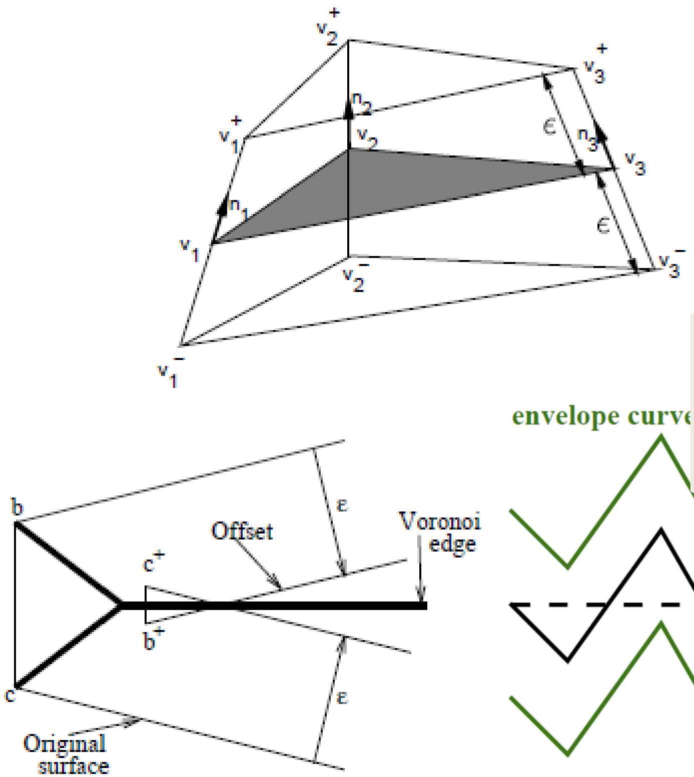


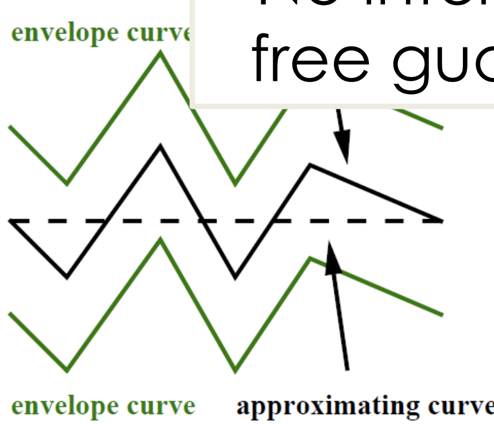


- **GPU-based Tolerance Volumes for Mesh Processing** [Botsch et al. 04]
  - Linear approximation of signed distance field

Tolerance Volume

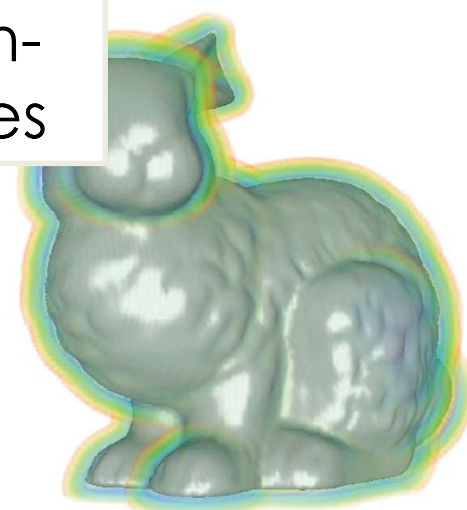
- **Simplification Envelopes** [Cohen et al 96]





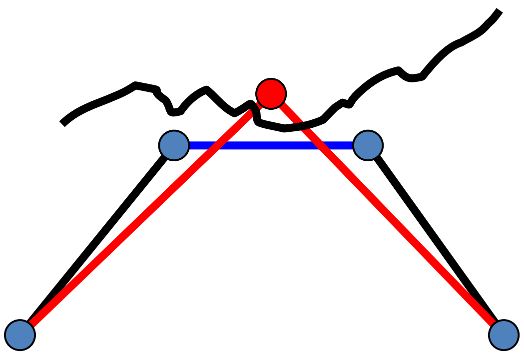
- **GPU-based Tolerance Volumes for Mesh Processing** [Botsch et al. 04]
  - Linear approximation of signed distance field
  - Fast marching to compute distance values and store as 3D distance texture

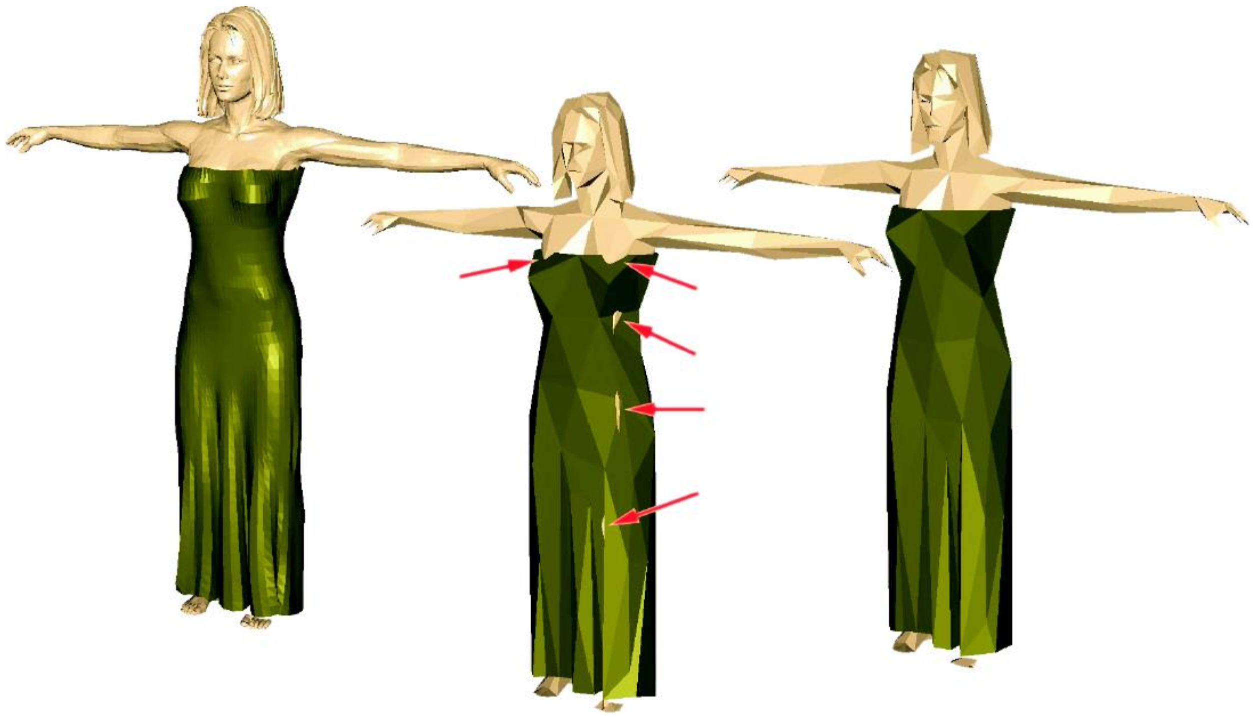
No intersection-free guarantees



Intersection Free

Intersection Free Simplification [Gumhold, Borodin, Klein], International Journal of Shape Modeling 2003.





Intersection Free

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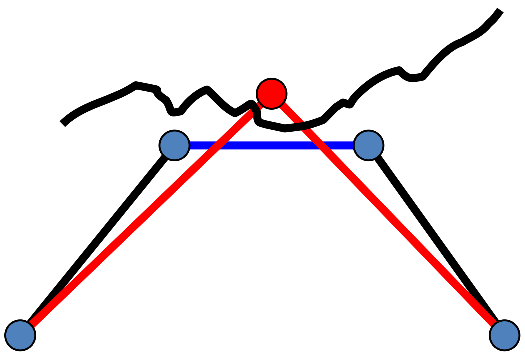
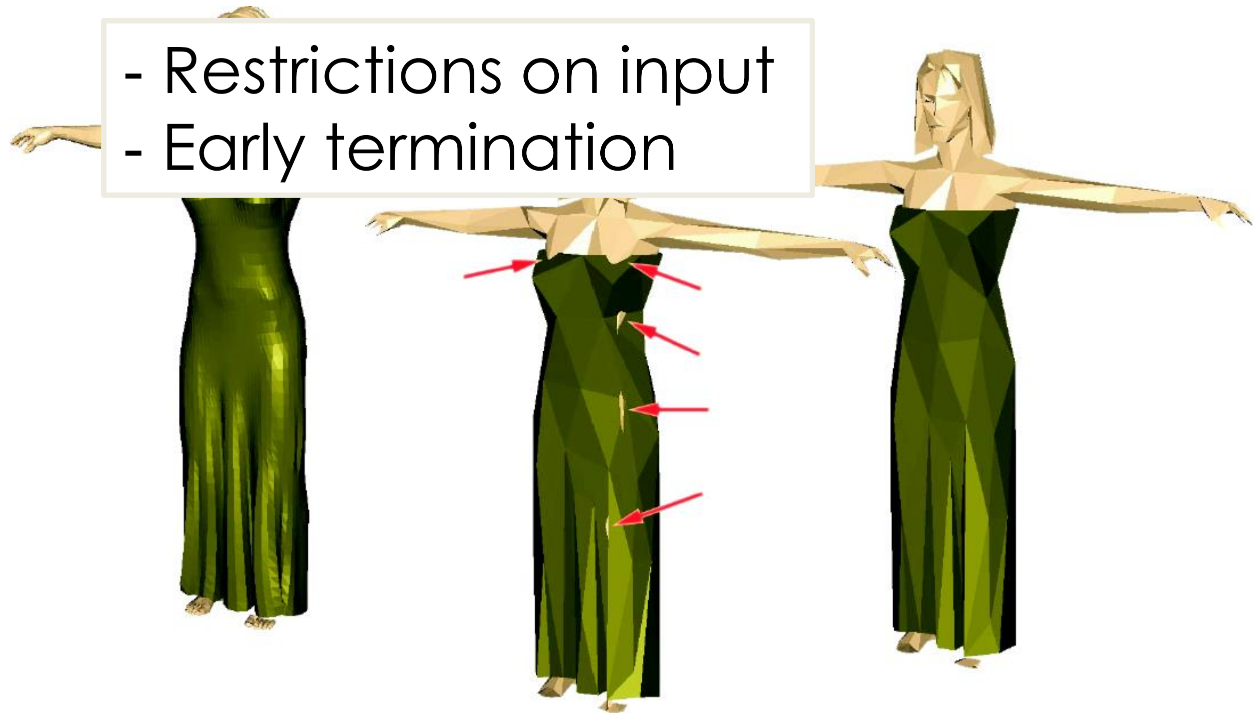
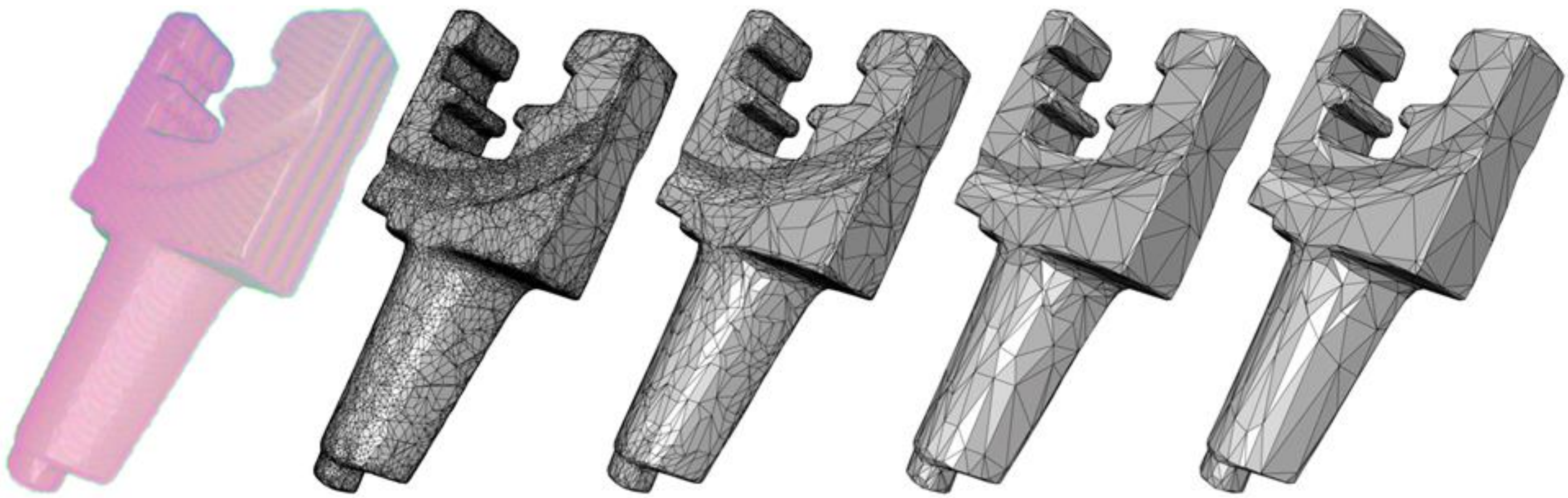


- Restrictions on input
- Early termination



Isotopic Approximation within a Tolerance Volume [Mandad, Cohen-Steiner, A.]



SIGGRAPH2015

Goals and Motivation

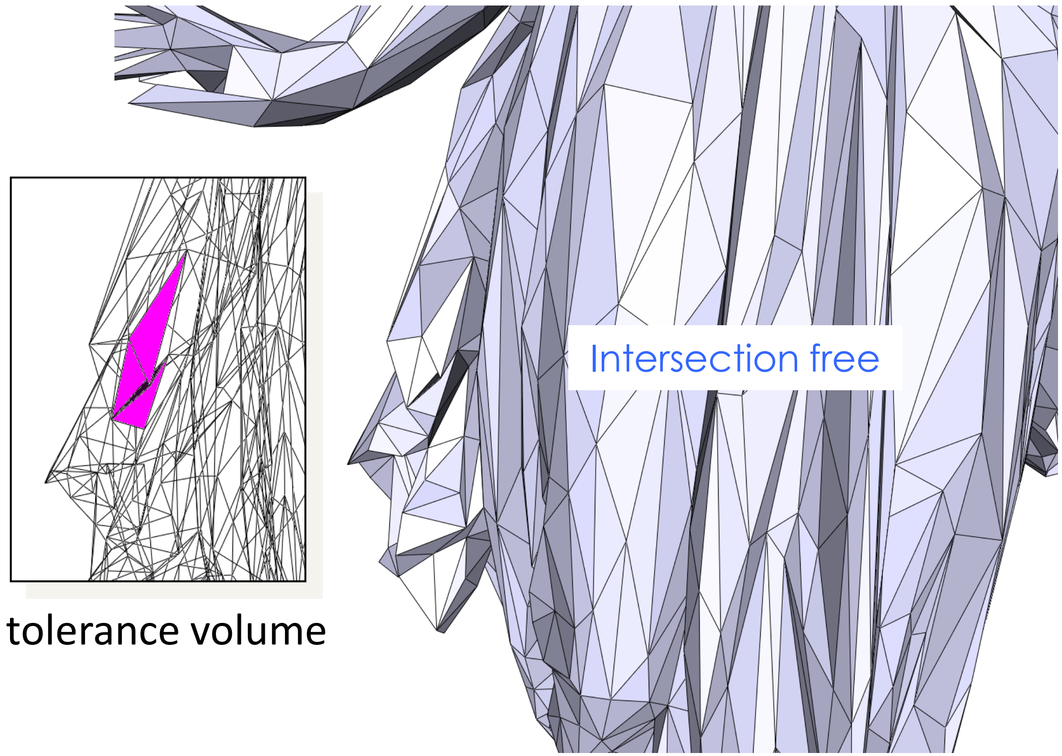
- **Input:**
  - Tolerance volume of a surface geometry
- **Output:**
  - Surface triangle mesh
  - Properties:
    - Within **tolerance volume**

Geometric Guarantee

Motivation: control global approximation error

Goals and Motivation

- **Input:**
  - Tolerance volume of a surface geometry
- **Output:**
  - Surface triangle mesh
  - Properties:
    - Within tolerance volume
    - **Homotopic equivalent** to tolerance volume

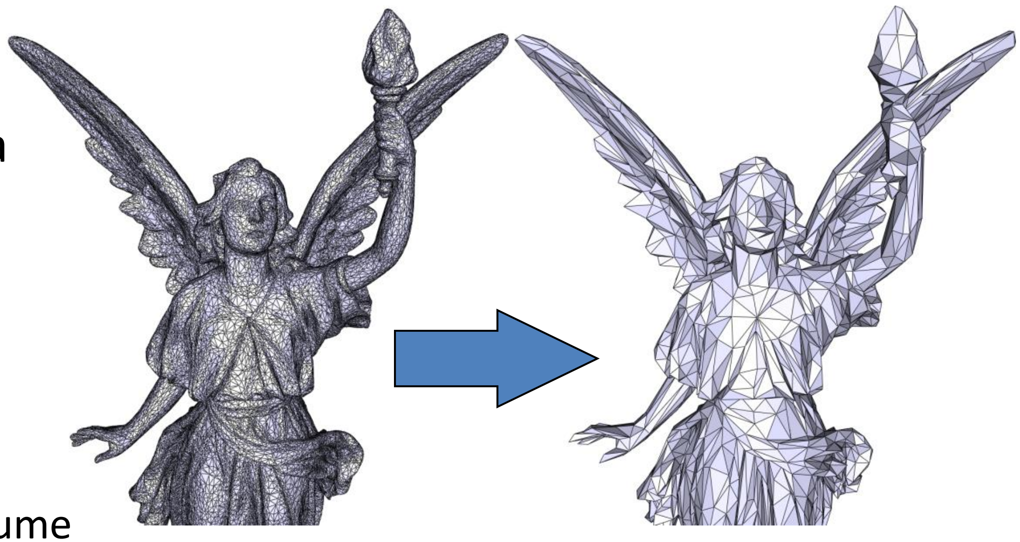


Topological Guarantee

Motivation: simulation, machining, printing etc.

Goals and Motivation

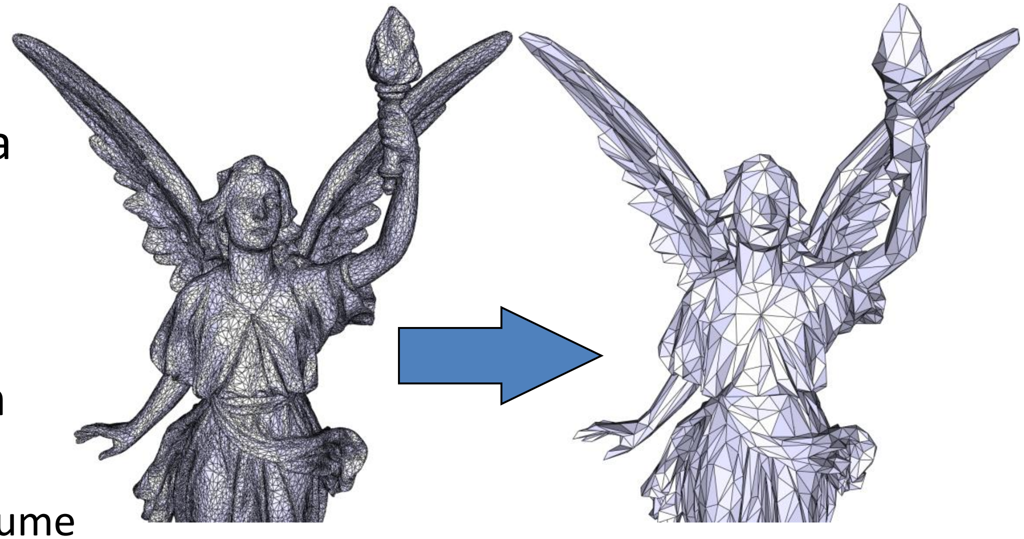
- **Input:**
  - Tolerance volume of a surface geometry
- **Output:**
  - Surface triangle mesh
  - Properties:
    - Within tolerance volume
    - Homotopic equivalent to tolerance volume
    - Minimum **number of vertices**



Motivation:
low triangle count

Goals and Motivation

- **Input:**
  - Tolerance volume of a surface geometry



- **Output:**
  - Surface triangle mesh
  - Properties:
    - Within tolerance volume
    - Homotopic equivalent to tolerance volume
      ~~Minimum **number of vertices**~~ NP-hard
    - Low **vertex count**

Motivation:
low triangle count

Approach

Starting Point

A Condition for Isotopic Approximation. Proceedings of ACM Symposium on Solid Modeling and Applications 2004.
Chazal, Cohen-Steiner.

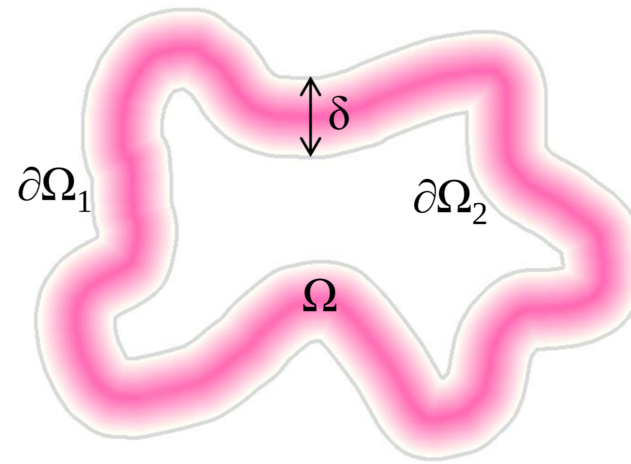
if

- S' is included in a topological thickening M of S
- S' separates sides of M
- S' is connected and its genus does not exceed the genus of S

Then, S' and S are isotopic.

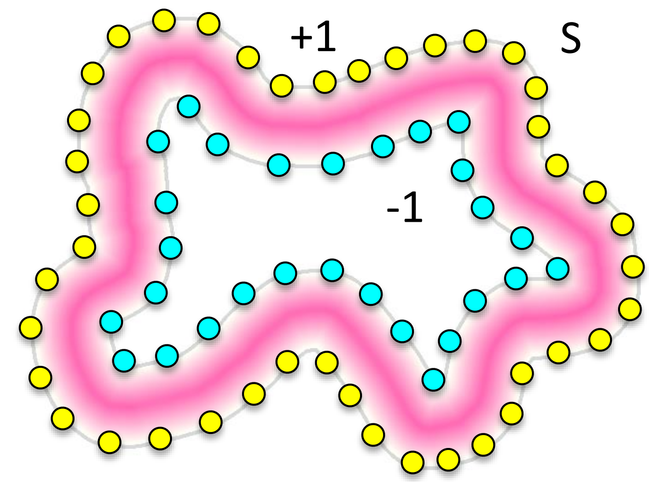
Approach

- **Input:** Tolerance Volume Ω



Approach

- **Input:** Tolerance Volume – Ω
- S : Point sample of ∂Ω with sampling σ
  - One **function value** (F) per connected component

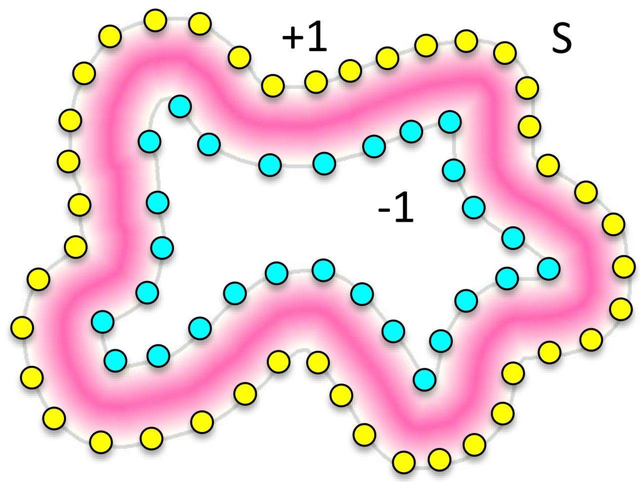


σ radii balls at
S cover ∂Ω

Approach

- **Input:** Tolerance Volume – Ω
- S : Point sample of ∂Ω with sampling σ
  - One **function value** (F) per connected component

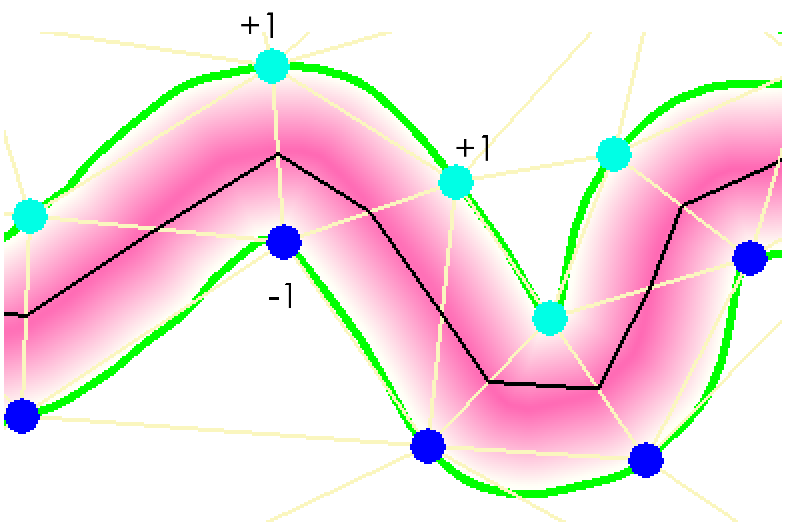
- Construct Triangulation on subset of S



Approach

- **Input:** Tolerance Volume – Ω
- S : Point sample of ∂Ω with sampling σ
  - One **function value** (F) per connected component

- Construct Triangulation on subset of S
- Piecewise linear function f on the triangulation interpolating **F**



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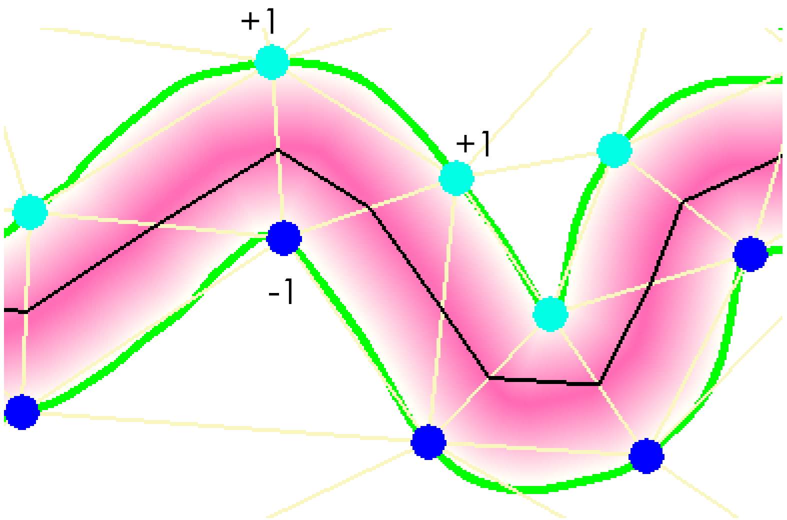


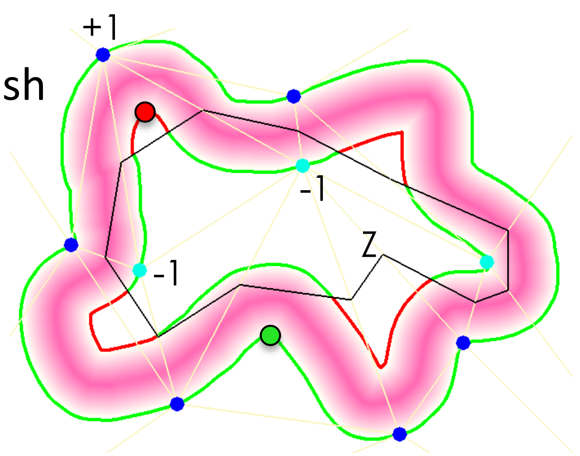
Zero-set separates tolerance boundaries

Approach

- **Input:** Tolerance Volume – Ω
- S : Point sample of ∂Ω with sampling σ
  - One **function value** (F) per connected component

- Construct Triangulation on subset of S
- Piecewise linear function f on the triangulation interpolating **F**
- Error: μ(s) = |F(s) – f(s)|
  - GOOD : μ(s) ≤ 1
  - BAD : otherwise

- Zero-set (Z) : closed intersection-free surface mesh



Approach

- **Input:** Tolerance Volume – Ω
- S : Point sample of ∂Ω with sampling σ
  - One **function value** (F) per connected component

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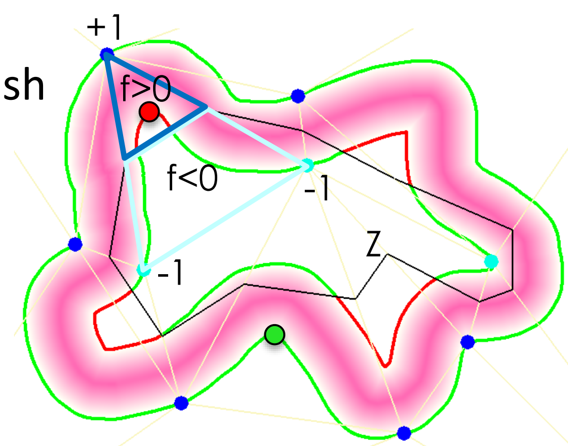

Approach

- **Input:** Tolerance Volume – Ω
- S : Point sample of ∂Ω with sampling σ
  - One **function value** (F) per connected component

- Construct Triangulation on subset of S
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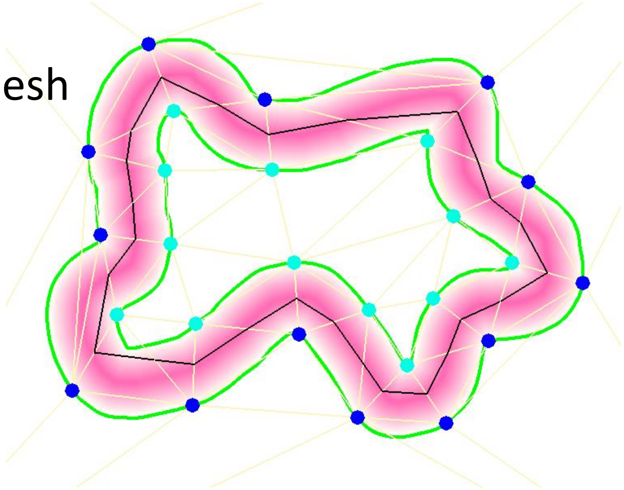
- Zero-set (Z) : closed intersection free surface mesh

- Simplification using edge collapse



Approach

- **Input:** Tolerance Volume – Ω
- S : Point sample of ∂Ω with sampling σ
  - One **function value** (F) per connected component

- Construct Triangulation on subset of S
- Piecewise linear function f on the triangulation interpolating **F**
- Error: $\mu(s) = |F(s) - f(s)|$
  - GOOD : $\mu(s) \leq 1$
  - BAD : otherwise

- Zero-set (Z) : closed intersection free surface mesh

Greedy Algorithm

- Simplification using edge collapse

Initialization	Refinement	Simplification

Approach

- **Input:** Tolerance Volume – Ω
- S : Point sample of ∂Ω with sampling σ
  - One **function value** (F) per connected component

- $\boxed{\text{Construct Triangulation on subset of S}}$
- Piecewise linear function f on the triangulation interpolating **F**
- Error: $\mu(s) = |F(s) - f(s)|$
  - GOOD : $\mu(s)_{\leq 1}$
  - BAD : otherwise
- $\boxed{\text{Zero-set (Z) :}}$ closed intersection free surface mesh

- Simplification using edge collapse

Greedy Algorithm

Coarse-to-Fine

Initialization	Refinement	Simplification

Approach

- **Input:** Tolerance Volume – Ω
- S : Point sample of ∂Ω with sampling σ
  - One **function value** (F) per connected component

- Construct Triangulation on subset of S
- Piecewise linear function f on the triangulation interpolating **F**
- Error: μ(s) = |F(s) – f(s)|
  - GOOD : μ(s) ≤ 1
  - BAD : otherwise
- Zero-set (Z) : closed intersection free surface mesh

- Simplification using edge collapse

Greedy Algorithm

Fine-to-Coarse

Initialization	Refinement	Simplification

Algorithm

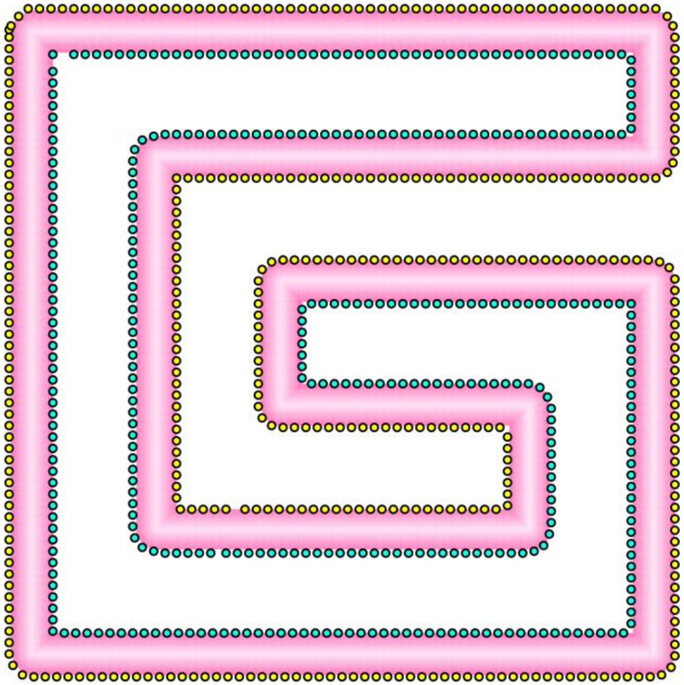
Init.

Refinement

Simplification

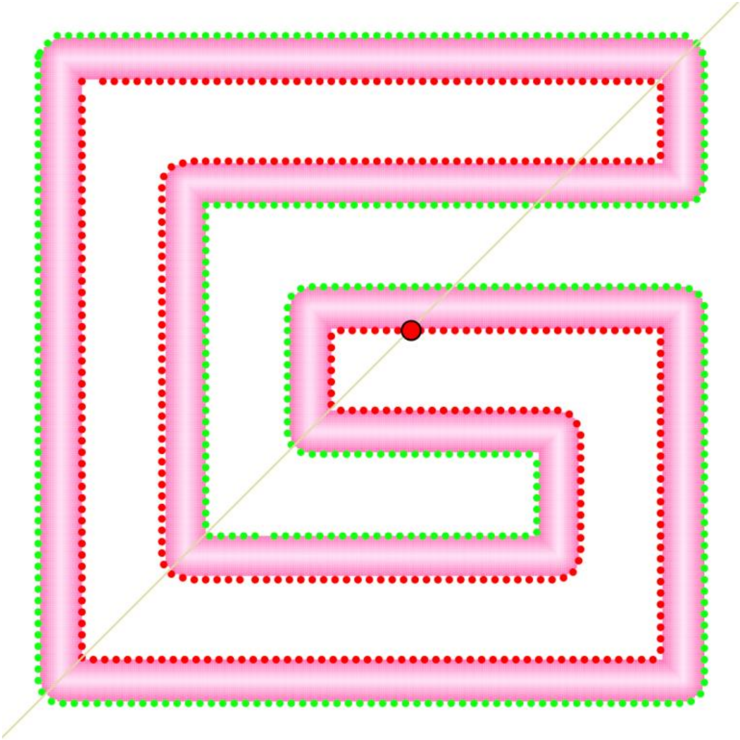
Algorithm (Initialization)

- Point sample of ∂Ω and label S



Algorithm (Initialization)

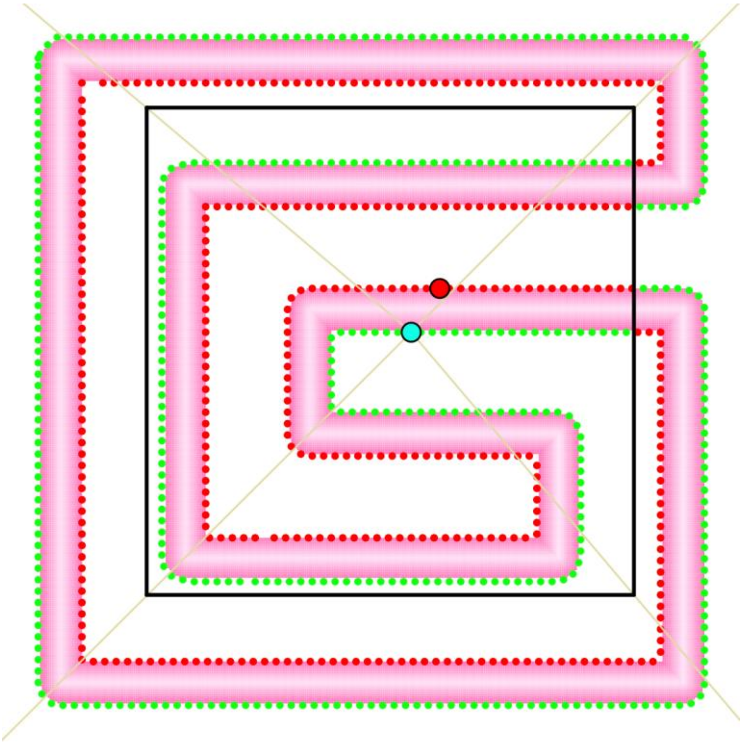
- Point sample of ∂Ω and label S
- Delaunay Triangulation T of loose bounding-box



Algorithm (coarse-to-fine)

- Point sample of ∂Ω and label S
- Delaunay Triangulation T of loose bounding-box

- While (**BAD** points)
  - Insert steiner point to T



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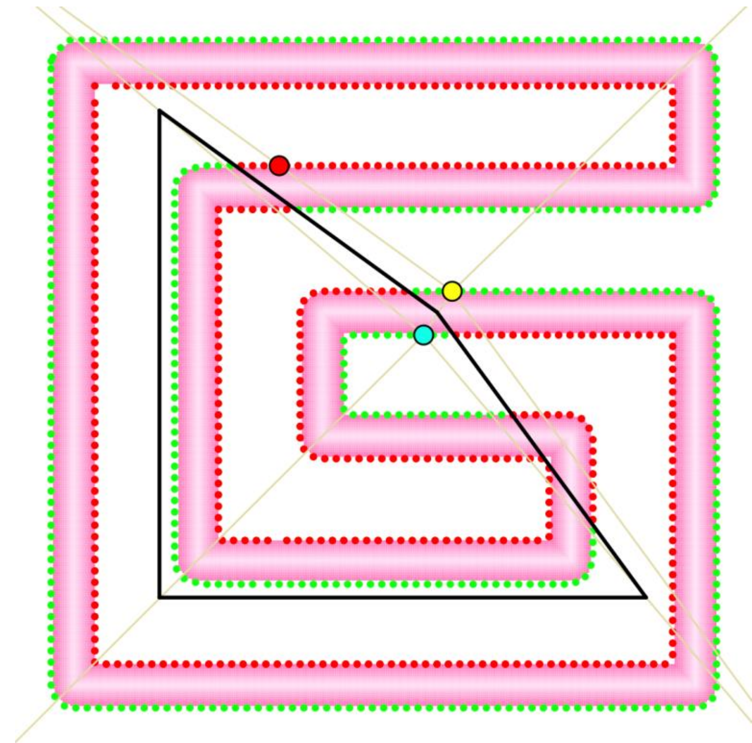

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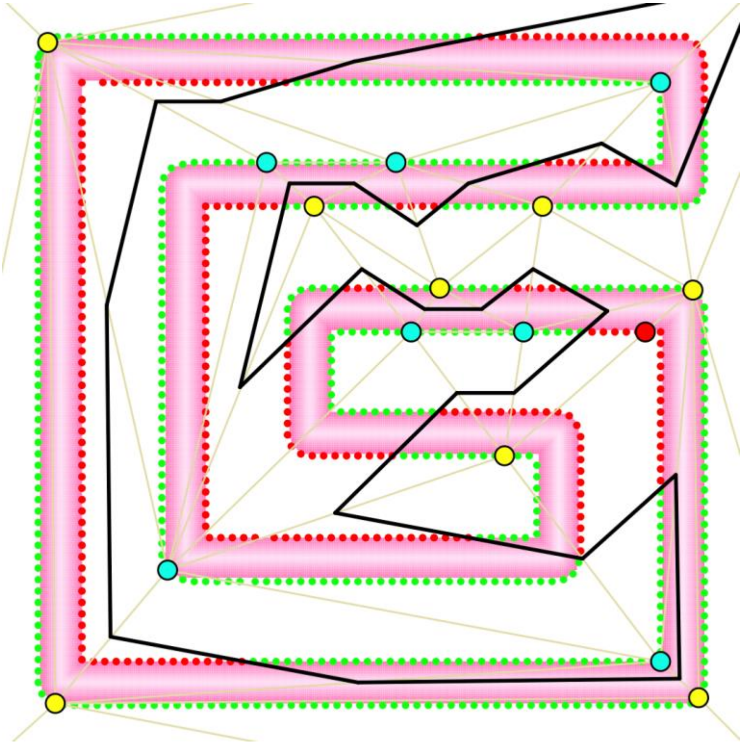
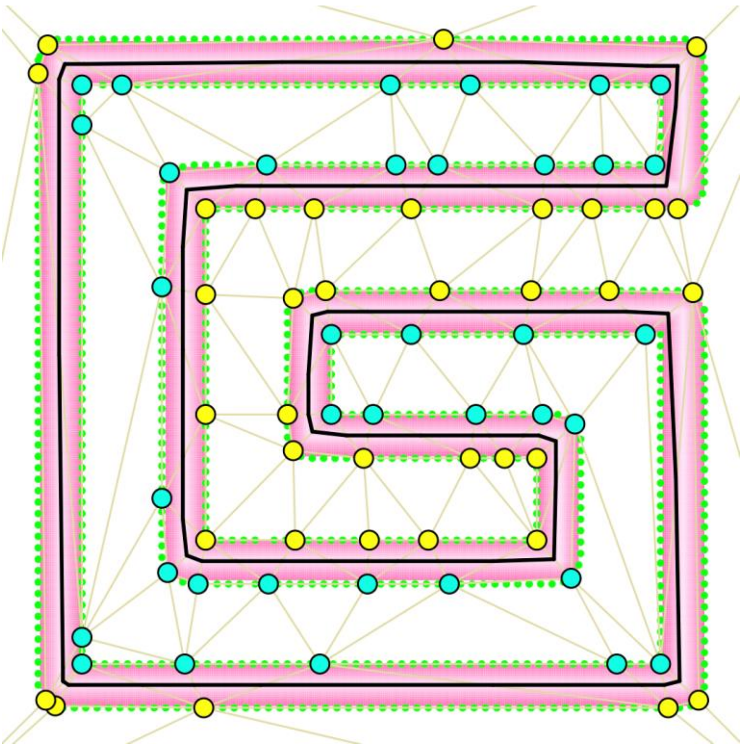


Algorithm (coarse-to-fine)

- Point sample of ∂Ω and label S
- Delaunay Triangulation T of loose bounding-box

- While (**BAD** points)
  - Insert steiner point to T



Init.

Refinement

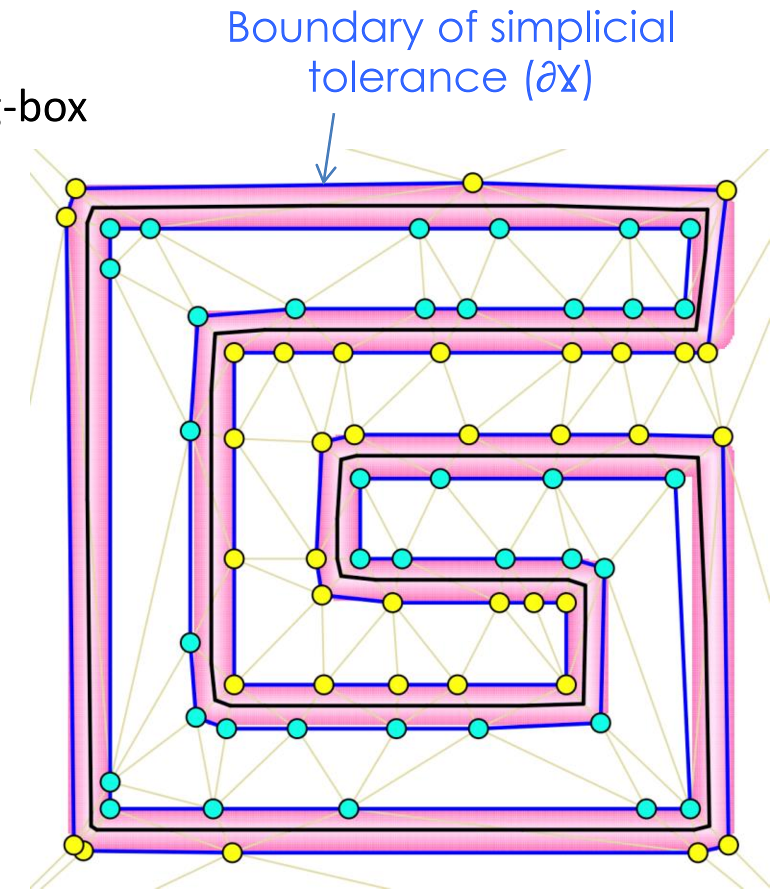
Simplification

Algorithm (coarse-to-fine)

- Point sample of ∂Ω and label S
- Delaunay Triangulation T of loose bounding-box
- While (**BAD** points)
  - Insert steiner point to T

topology

anisotropy



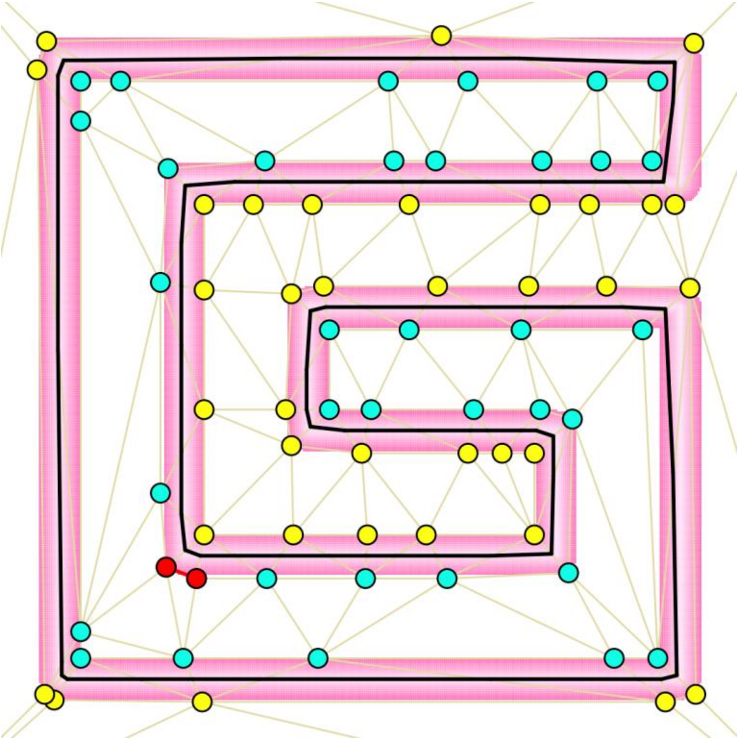
Algorithm (fine-to-coarse)

Delaunay → Non Delaunay

- Point sample of ∂Ω and label S
- Delaunay Triangulation T of loose bounding-box

- While (**BAD** points)
  - Insert steiner point to T

- Collapse edges of ∂Ӿ
(preserving classification of S)

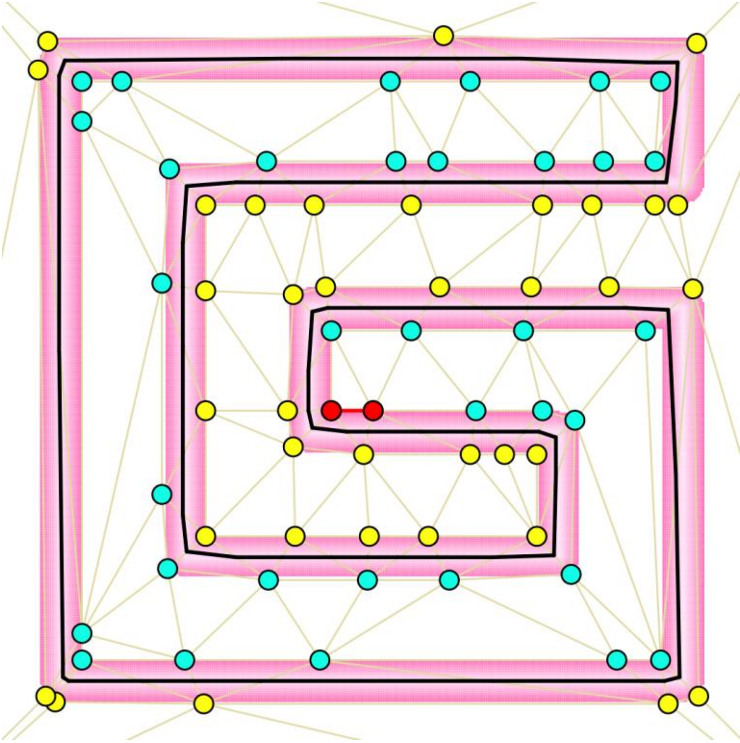


Algorithm (fine-to-coarse)

- Point sample of ∂Ω and label S
- Delaunay Triangulation T of loose bounding-box

- While (**BAD** points)
  - Insert steiner point to T

- Collapse edges of ∂Ӿ
(preserving classification of S)

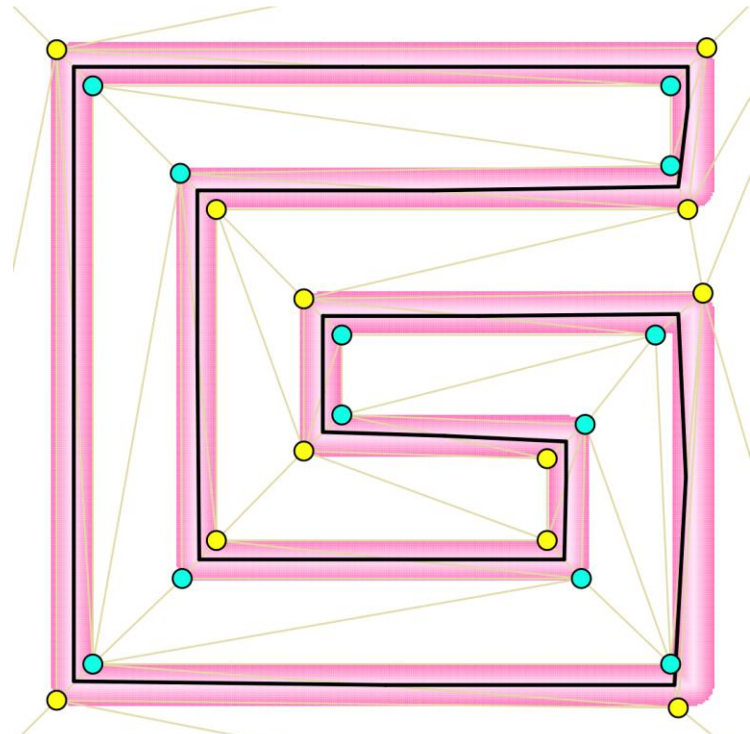


Algorithm (fine-to-coarse)

- Point sample of ∂Ω and label S
- Delaunay Triangulation T of loose bounding-box

- While (**BAD** points)
  - Insert steiner point to T

- Collapse edges of ∂Ӿ
(preserving classification of S)



Init.

Refinement

Simplification

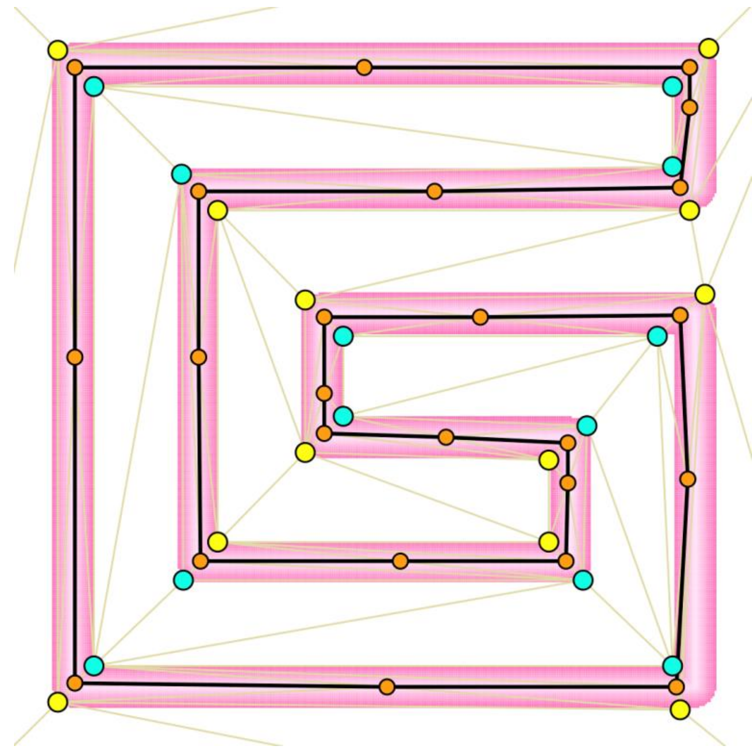
Algorithm (fine-to-coarse)

- Point sample of ∂Ω and label S
- Delaunay Triangulation T of loose bounding-box

- While (**BAD** points)
  - Insert steiner point to T

- Collapse edges of ∂X
  (preserving classification of S)

- Insert Z in T



Init.

Refinement

Simplification

Algorithm (fine-to-coarse)

- Point sample of ∂Ω and label S
- Delaunay Triangulation T of loose bounding-box

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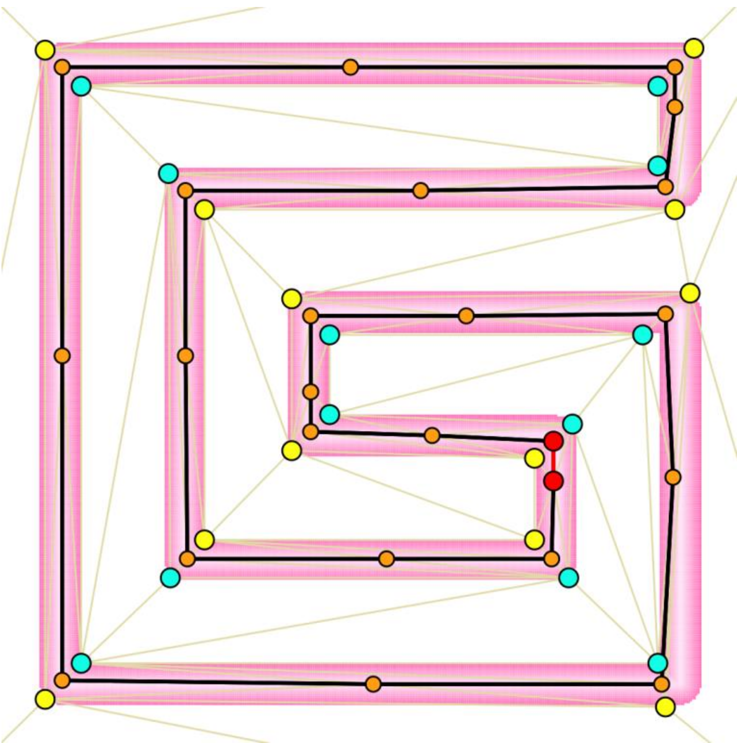
- Insert Z in T
- Collapse edges of Z



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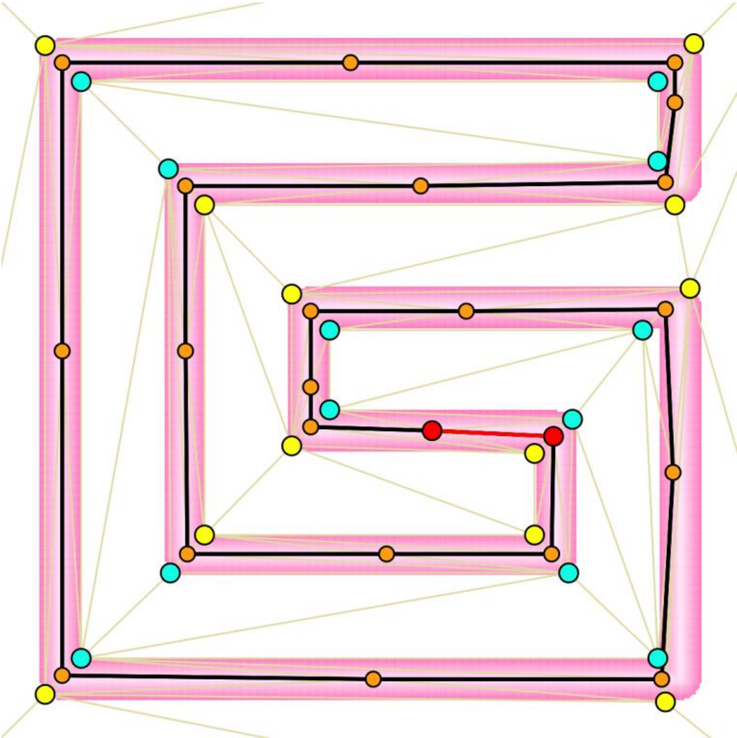

Algorithm (fine-to-coarse)

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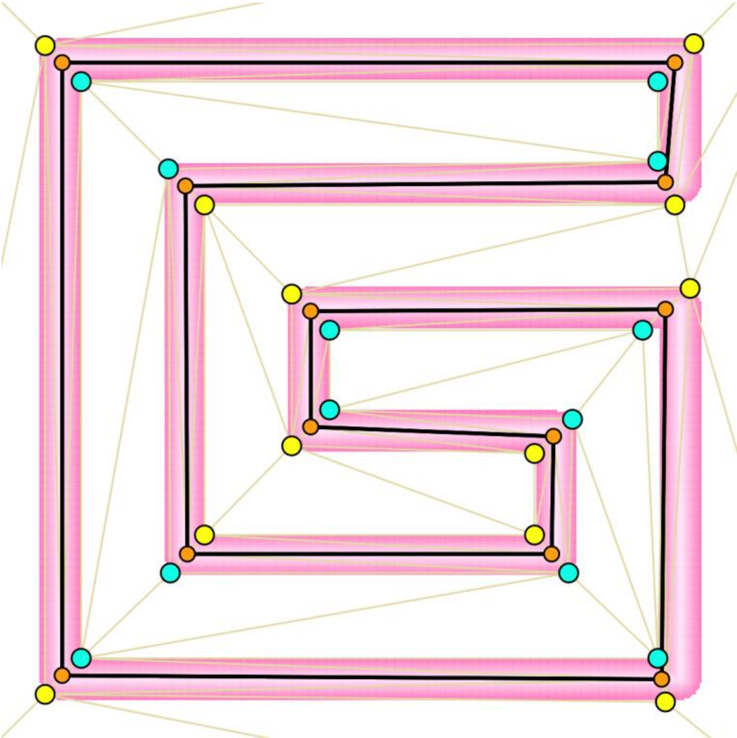
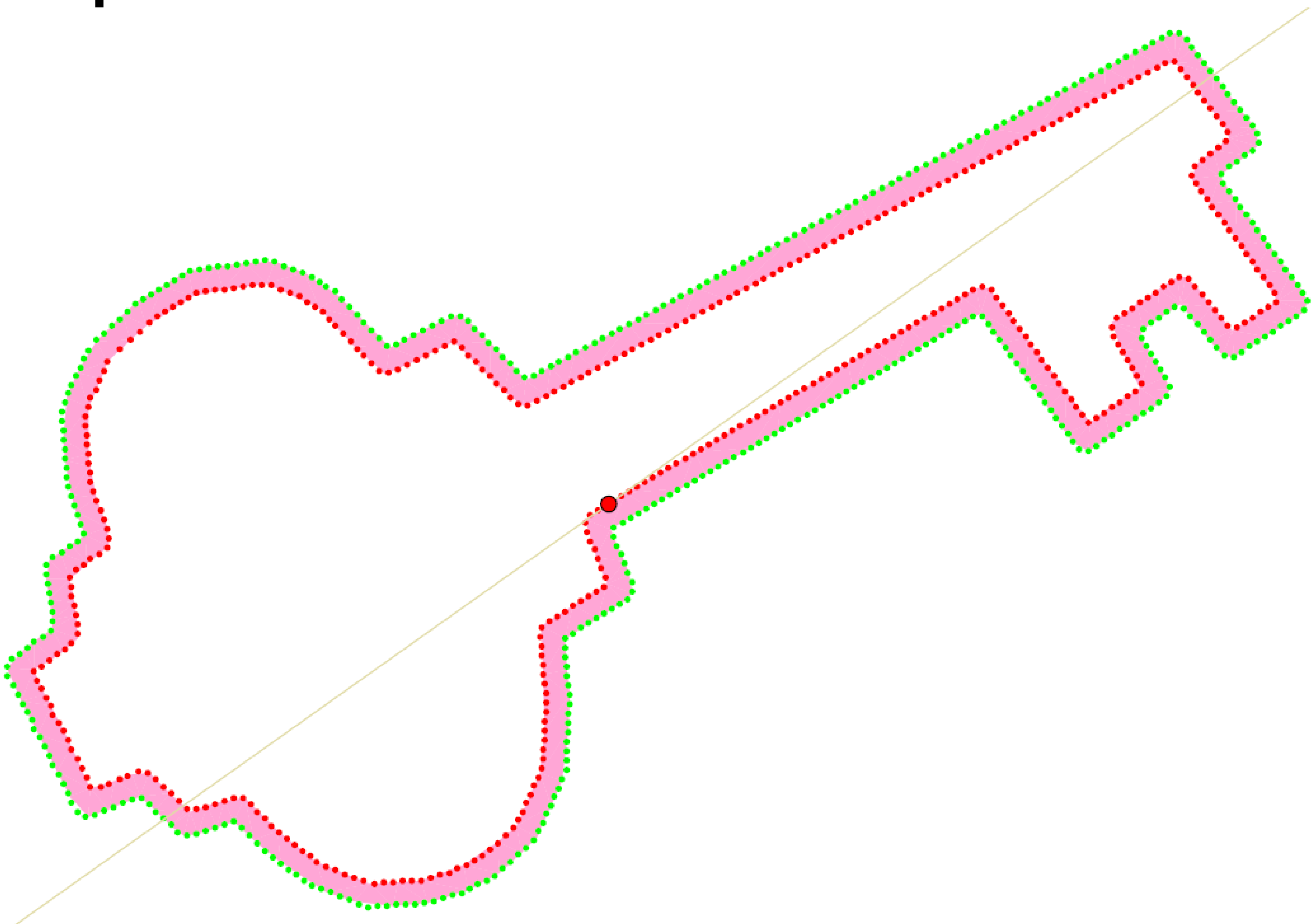
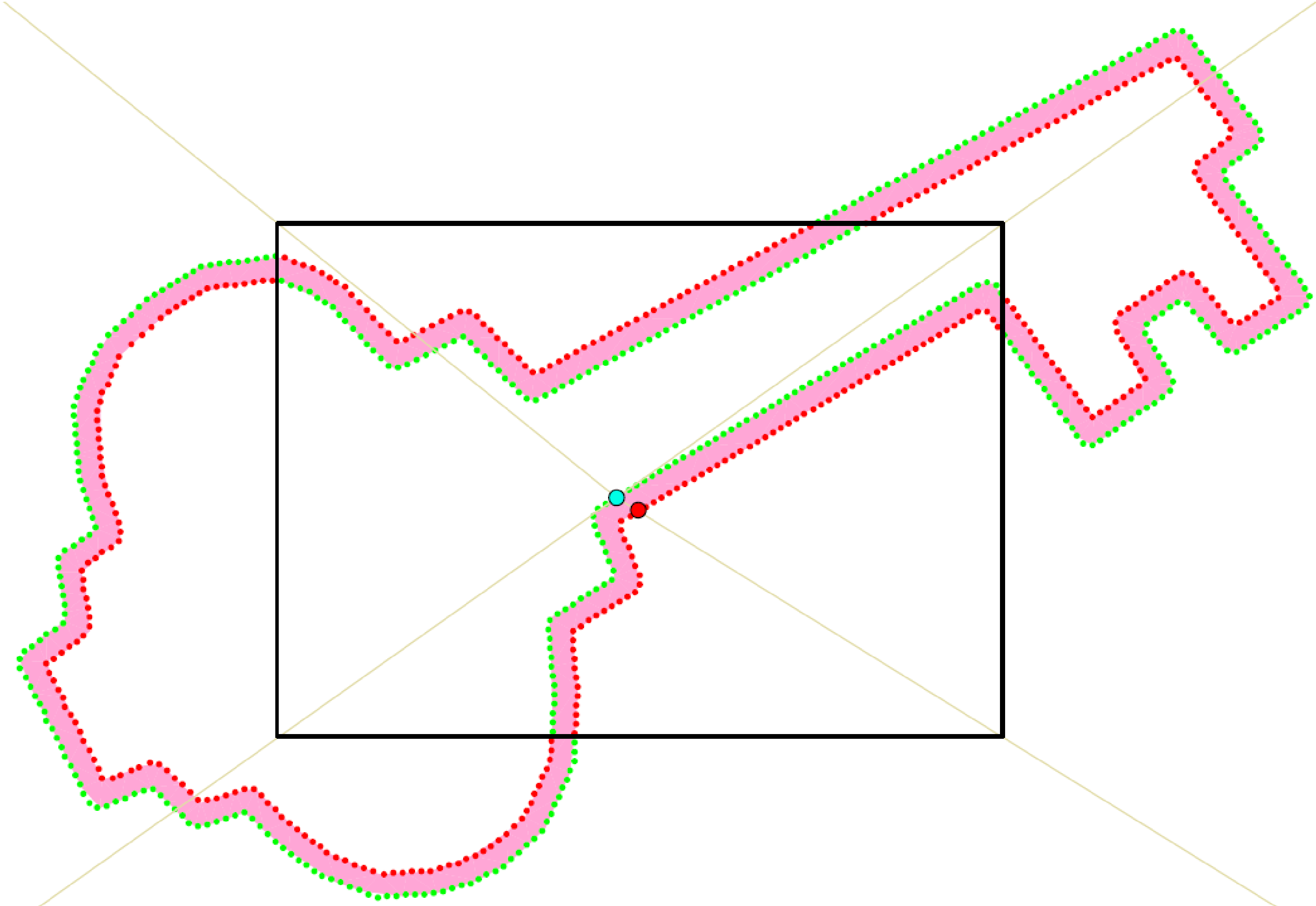


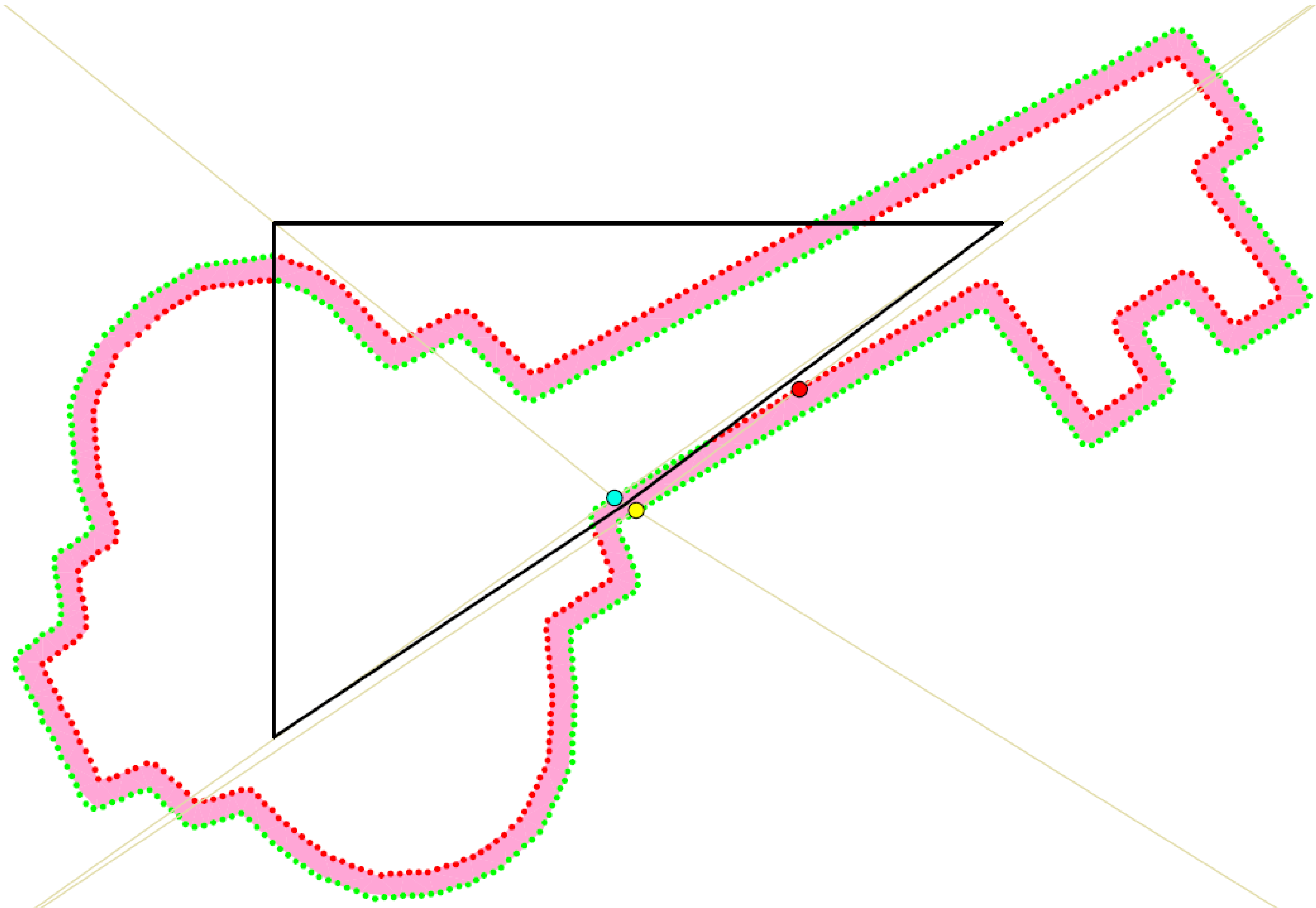
Recap



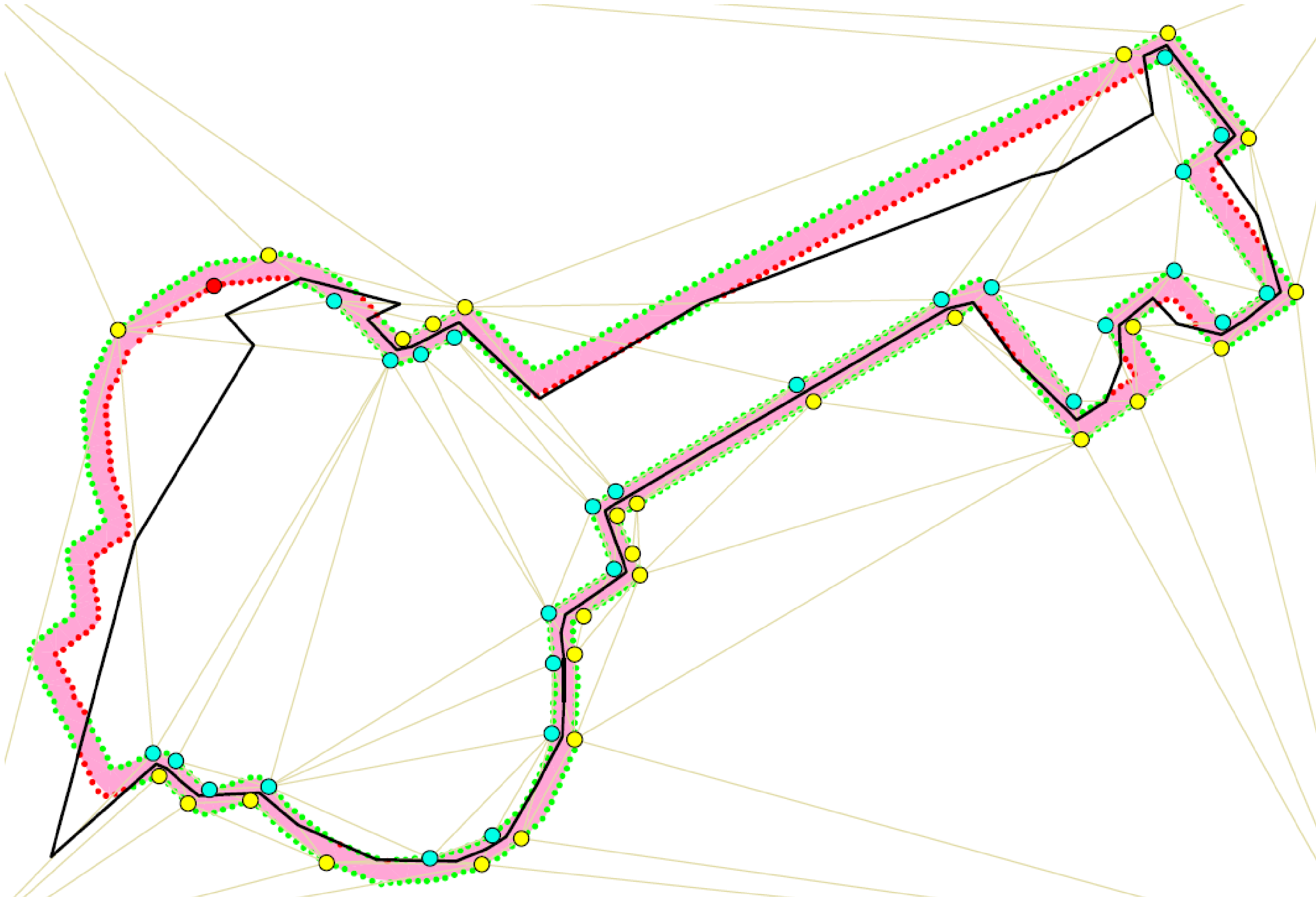
Recap



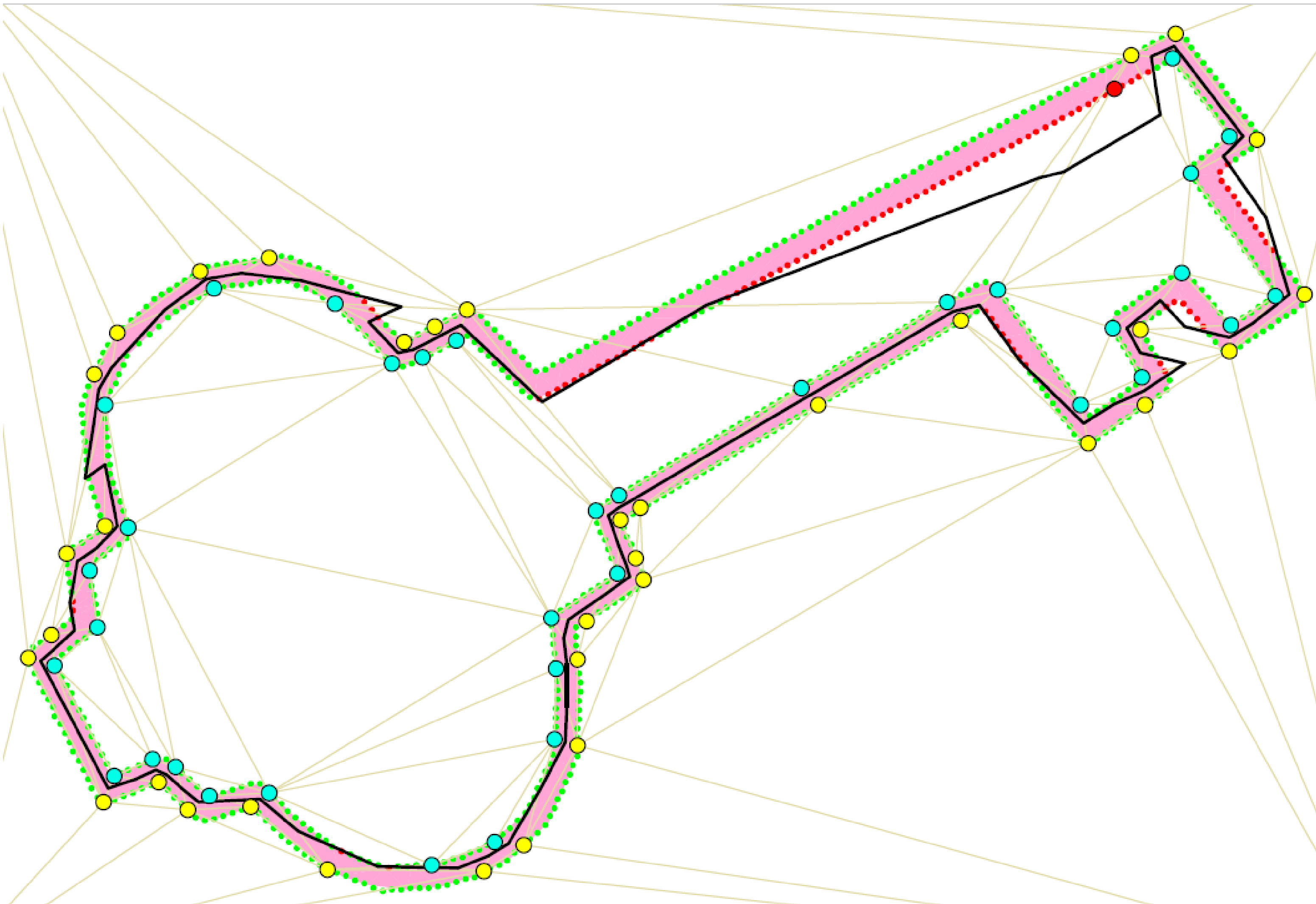
Recap



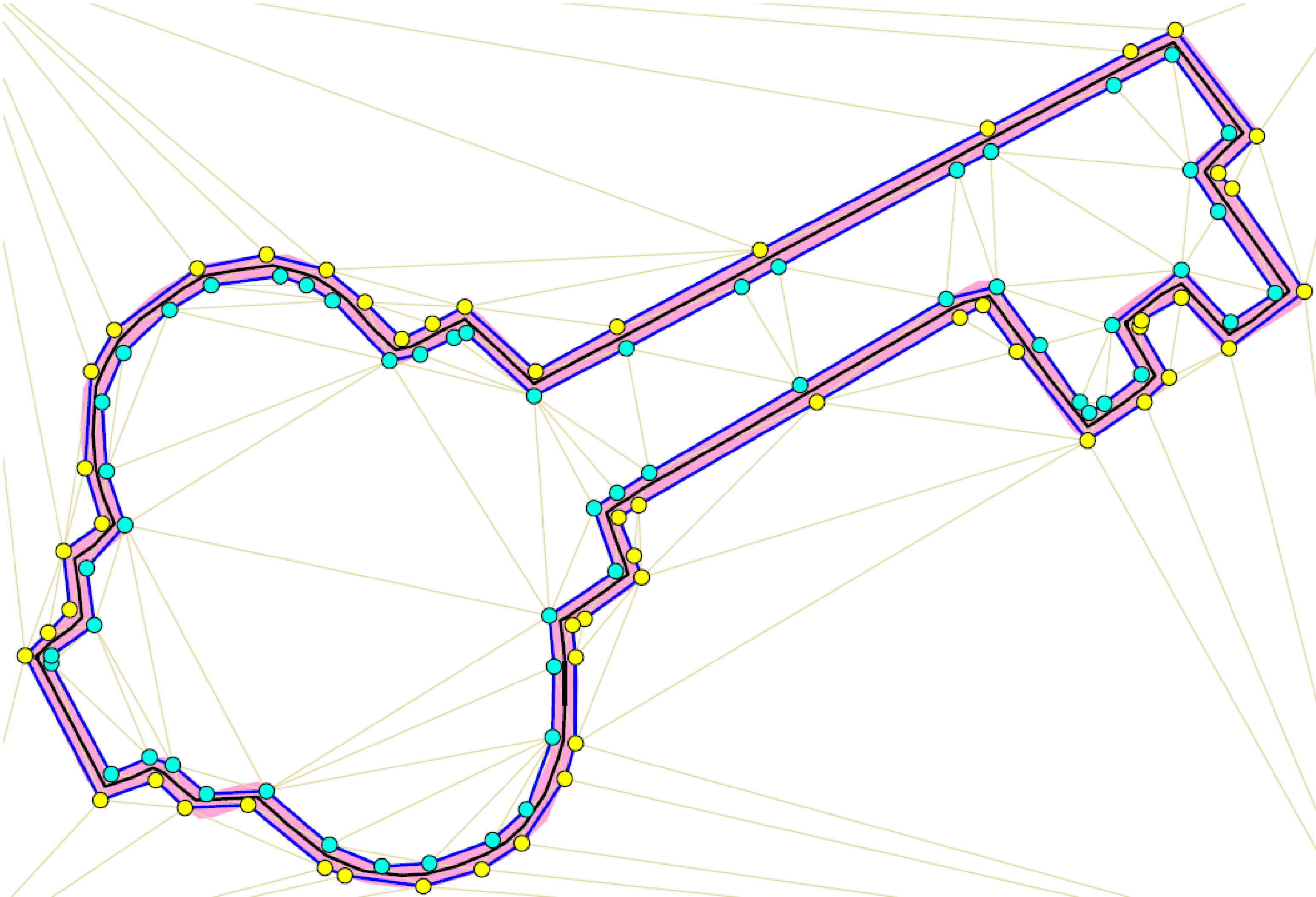
Recap



Recap



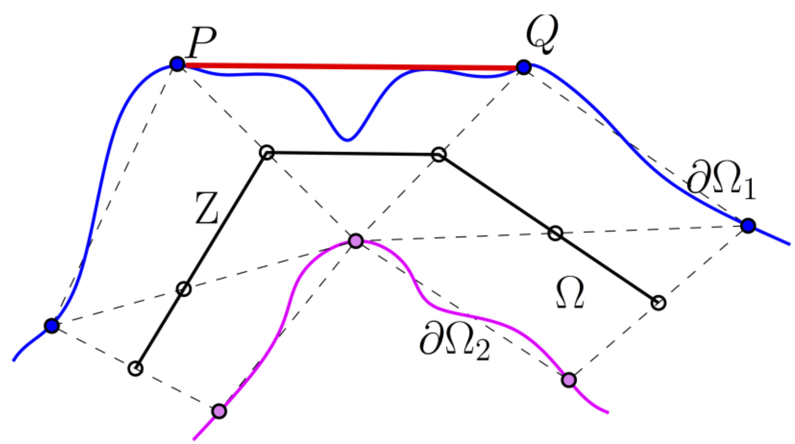
Recap



Simplification

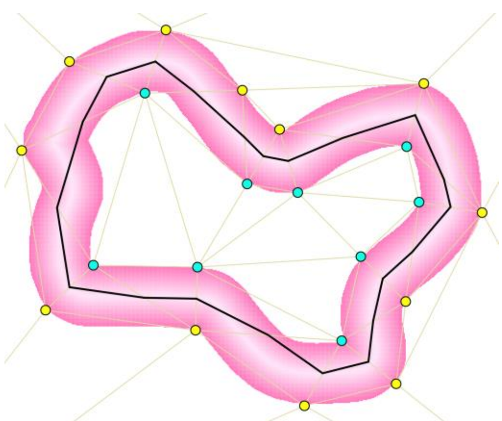
Simplification (Multiple stages)

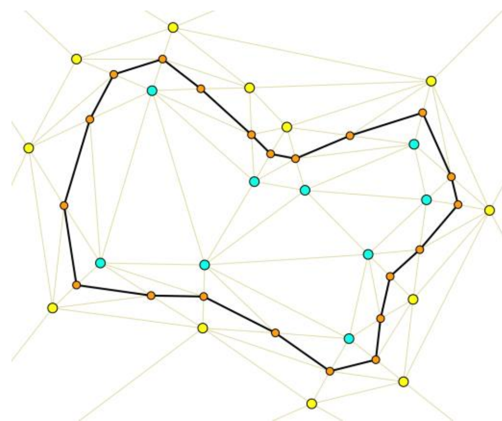
- Edge collapse on ∂X



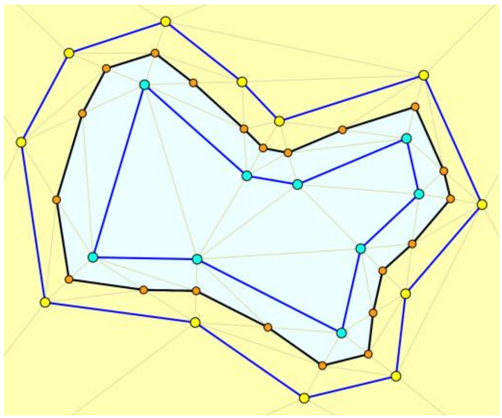
Simplification (Multiple stages)

- Edge collapse on ∂X
- Mutual Tessellation





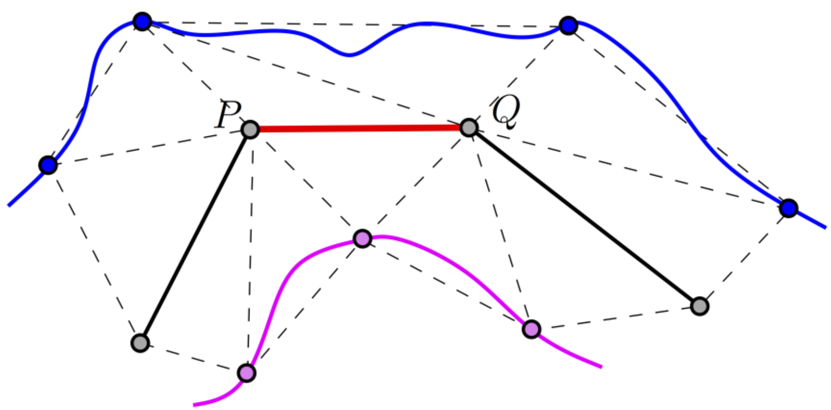
Mutual Tessellation



Classification of simplices based on $\partial\Omega_i$

Simplification (Multiple stages)

- Edge collapse on ∂X
- Mutual Tessellation
- Edge collapse on Z



Init. Refinement Simplification

Simplification (Multiple stages)

- Edge collapse on $\partial\mathcal{X}$
- Mutual Tessellation
- Edge collapse on Z
- Edge collapse between $\mathcal{X}$ and Z which may induce further edge collapses of Z

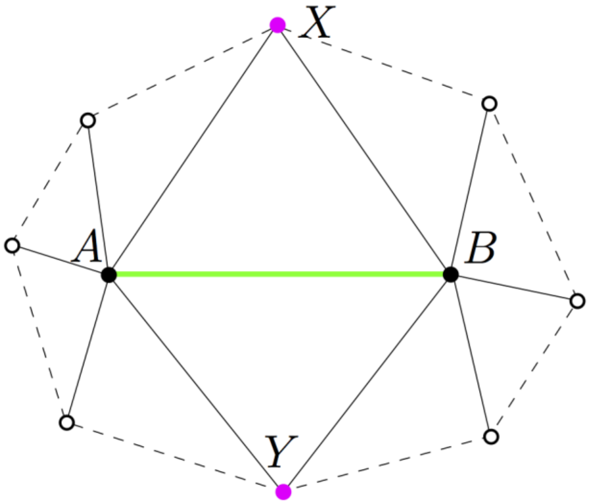
Simplification

- Edge collapse?

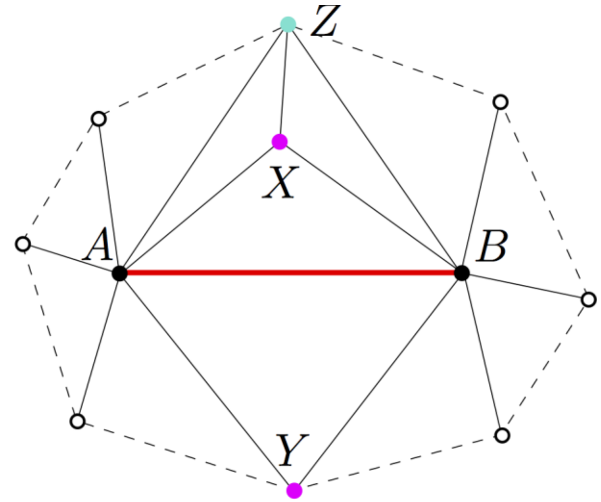
Simplification (conditions)

- Validity of Triangulation
  - Combinatorial Topology [Dey et al 1998]

Lk(AB) = Lk(A) ∩ Lk(B)



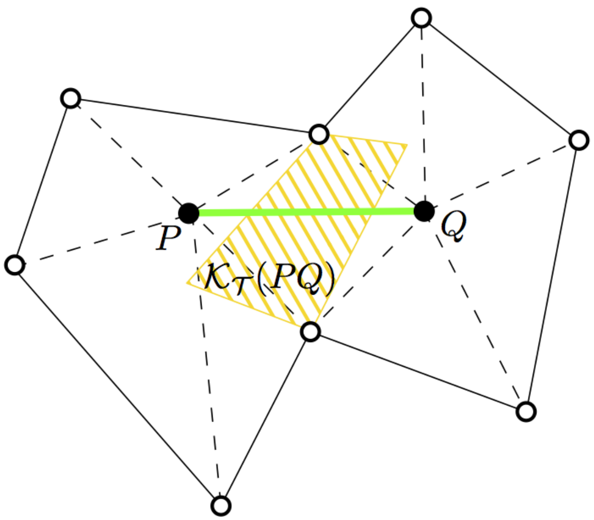
Lk(AB) ≠ Lk(A) ∩ Lk(B)



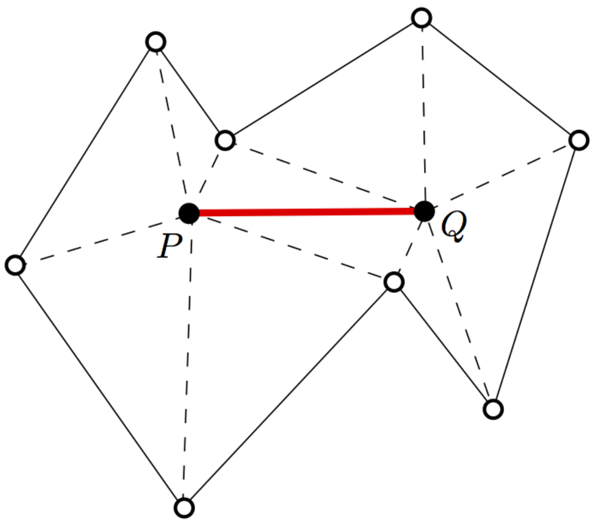
Combinatorial topology preserved upon collapse of edge AB

Simplification (conditions)

- Validity of Triangulation
  - Combinatorial Topology [Dey et al 1998]
  - Valid Embedding – Kernel of 1-ring



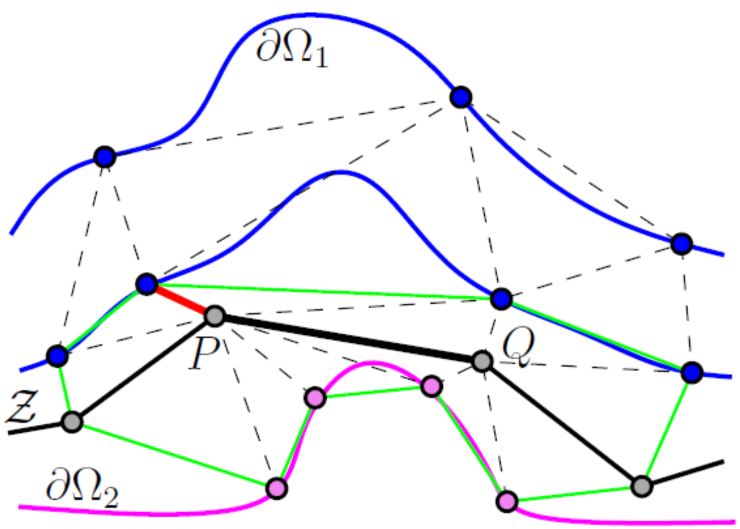
Edge PQ collapsible within kernel



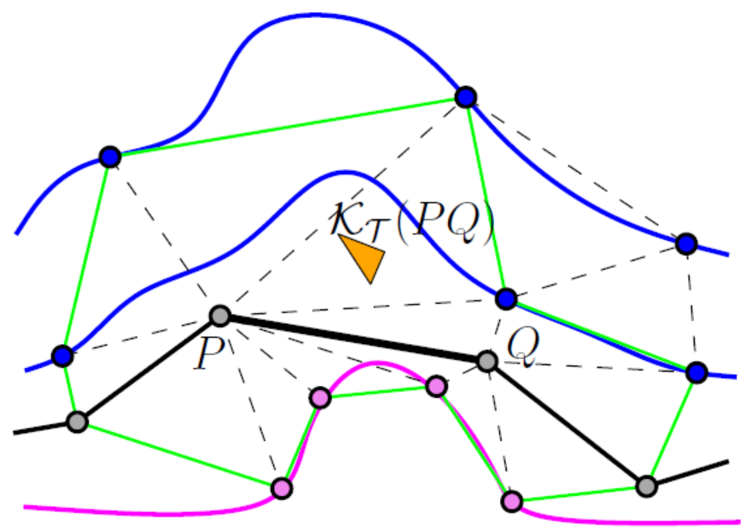
Empty kernel

Simplification (conditions)

- Validity of Triangulation
  - Combinatorial Topology [Dey et al 1998]
  - Valid Embedding – Kernel of 1-ring



empty



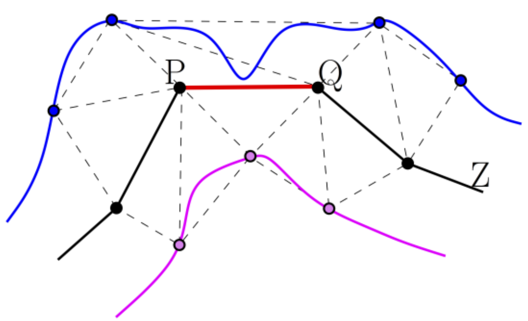
non-empty

Simplification (conditions)

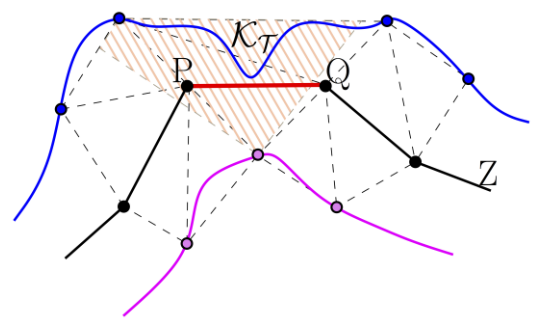
- Validity of Triangulation
  - Combinatorial Topology [Dey et al 1998]
  - Valid Embedding – Kernel of 1-ring

- Preserve classification of S – Non convex problem

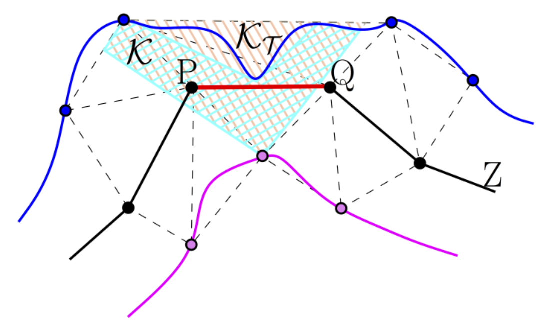
Point sampling of kernel K_T during simulation of edge collapse



Edge PQ to be collapsed



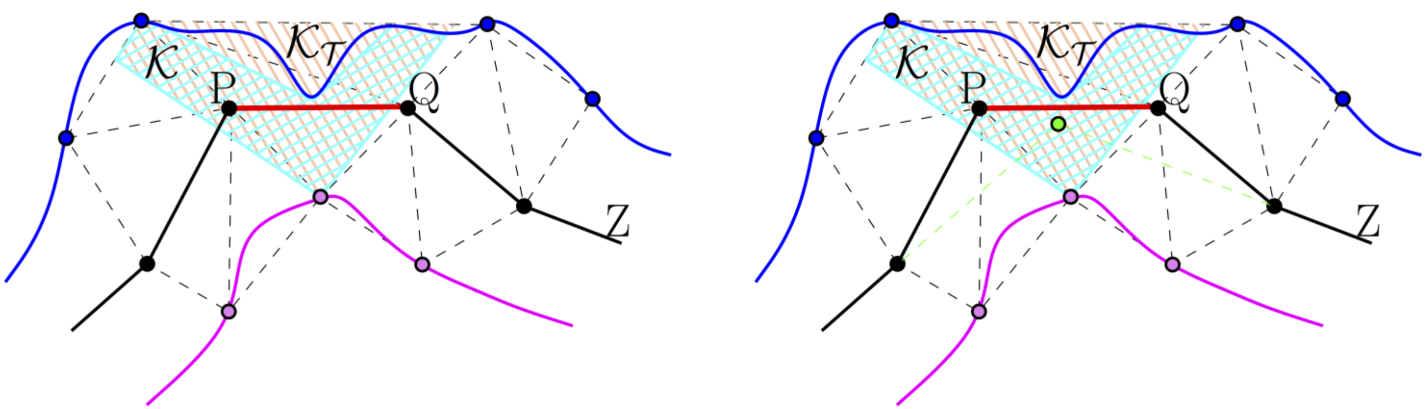
Valid Embedding kernel



Kernel to preserve Classification

Simplification (conditions)

- Validity of Triangulation
  - Combinatorial Topology [Dey et al 1998]
  - Valid Embedding – Kernel of 1-ring
- Preserve classification of S – Non convex problem
- Minimize error: Sum of squared distance between target vertex and 2-ring planes



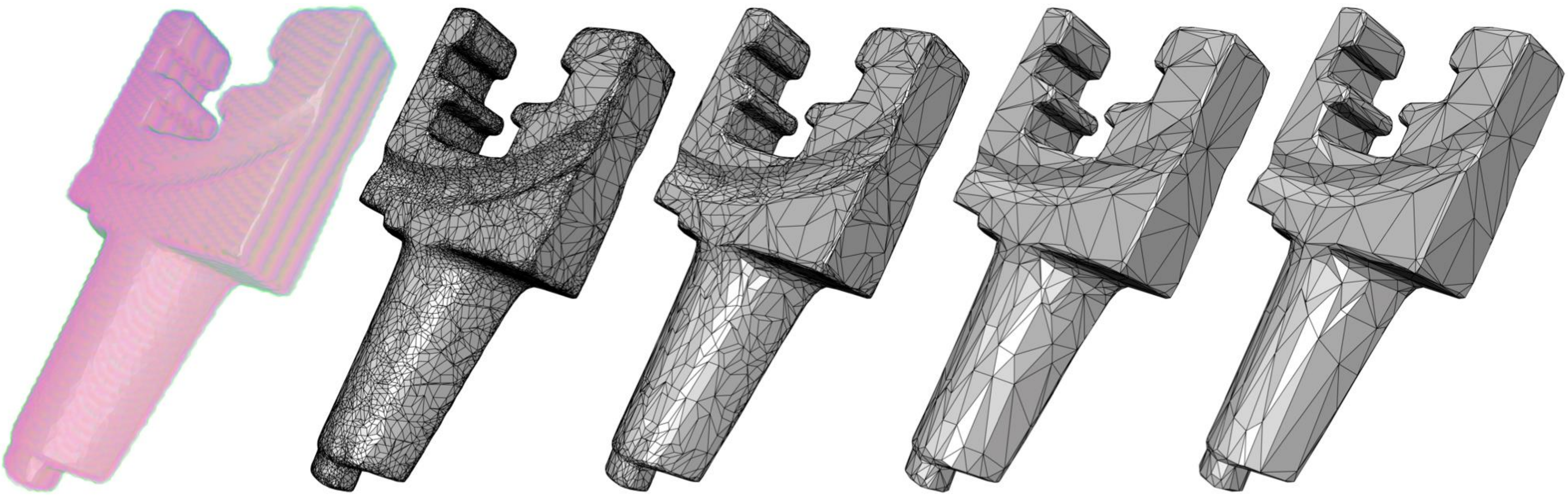
Init.
Refinement
Simplification

Simplification (conditions)

- Validity of Triangulation
  - Combinatorial Topology [Dey et al 1998]
  - Valid Embedding – Kernel of 1-ring
- Preserve classification of S – Non convex problem
- Minimize error: Sum of squared distance between target vertex and 2-ring planes
- Faithful normals (same as in refinement)

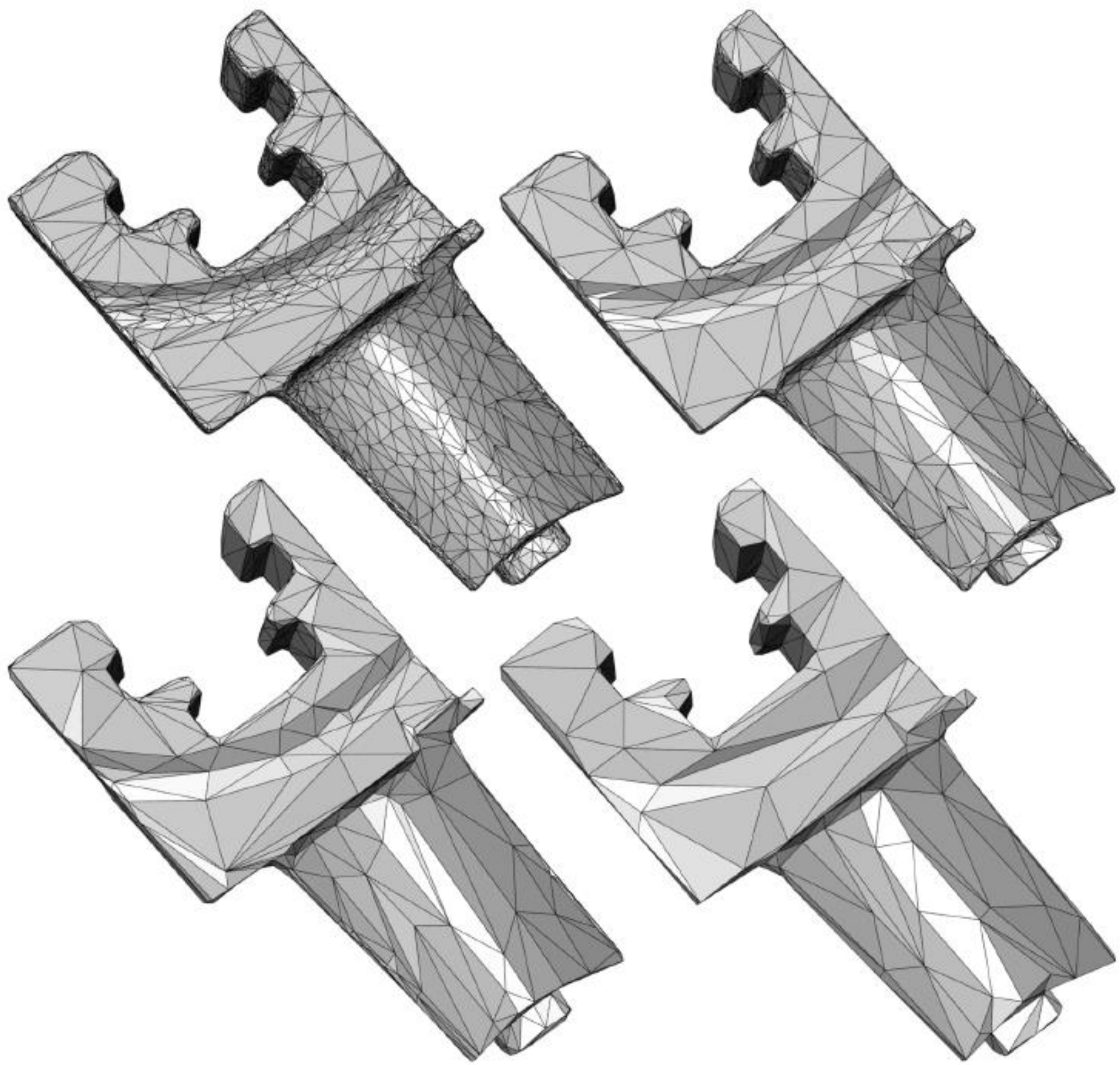
Results

Blade (steps)

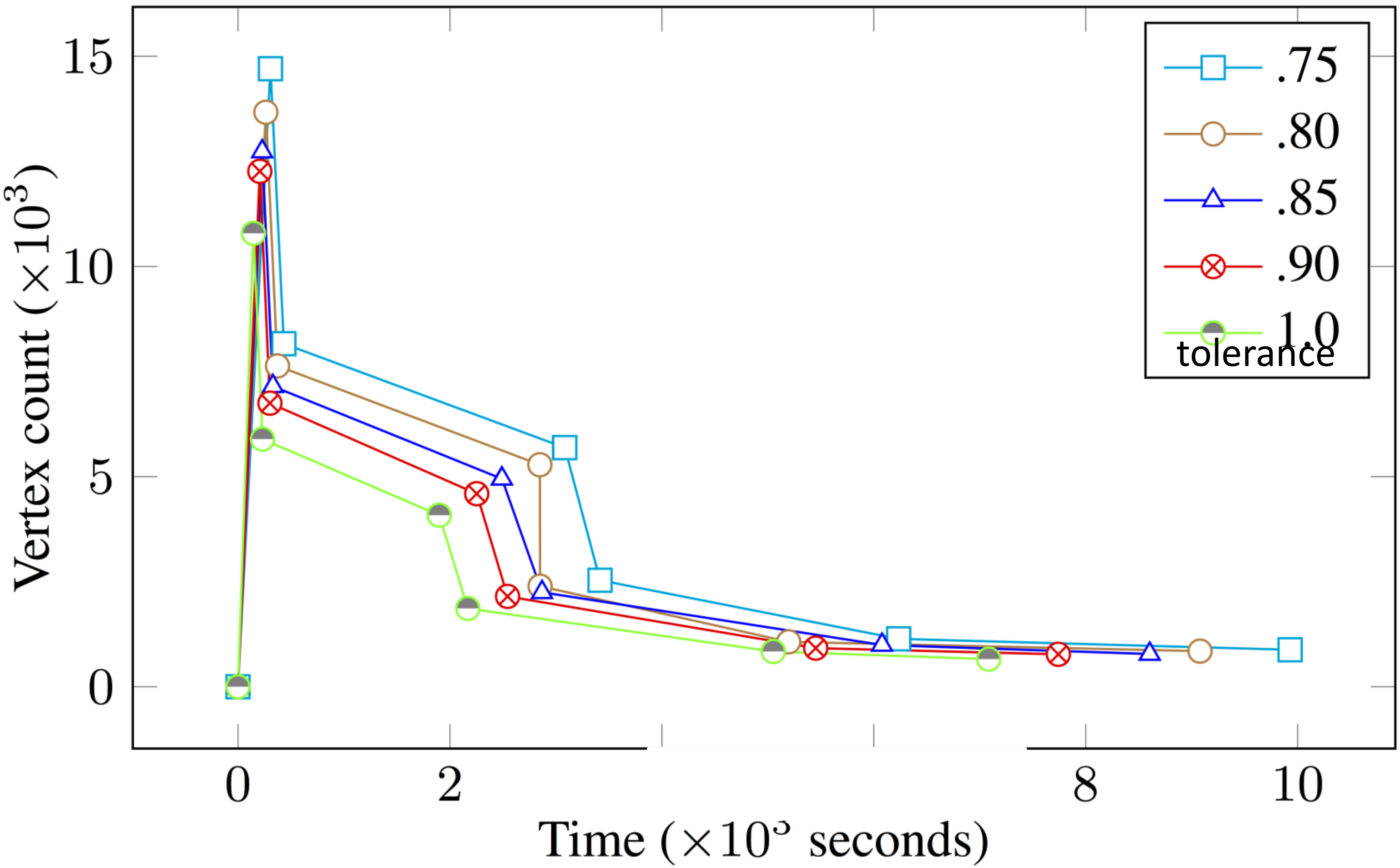


Input Ω | Refinement | Simplification on ∂X | Simplification on Z | Simplification on all edges

Blade (varying tolerance)

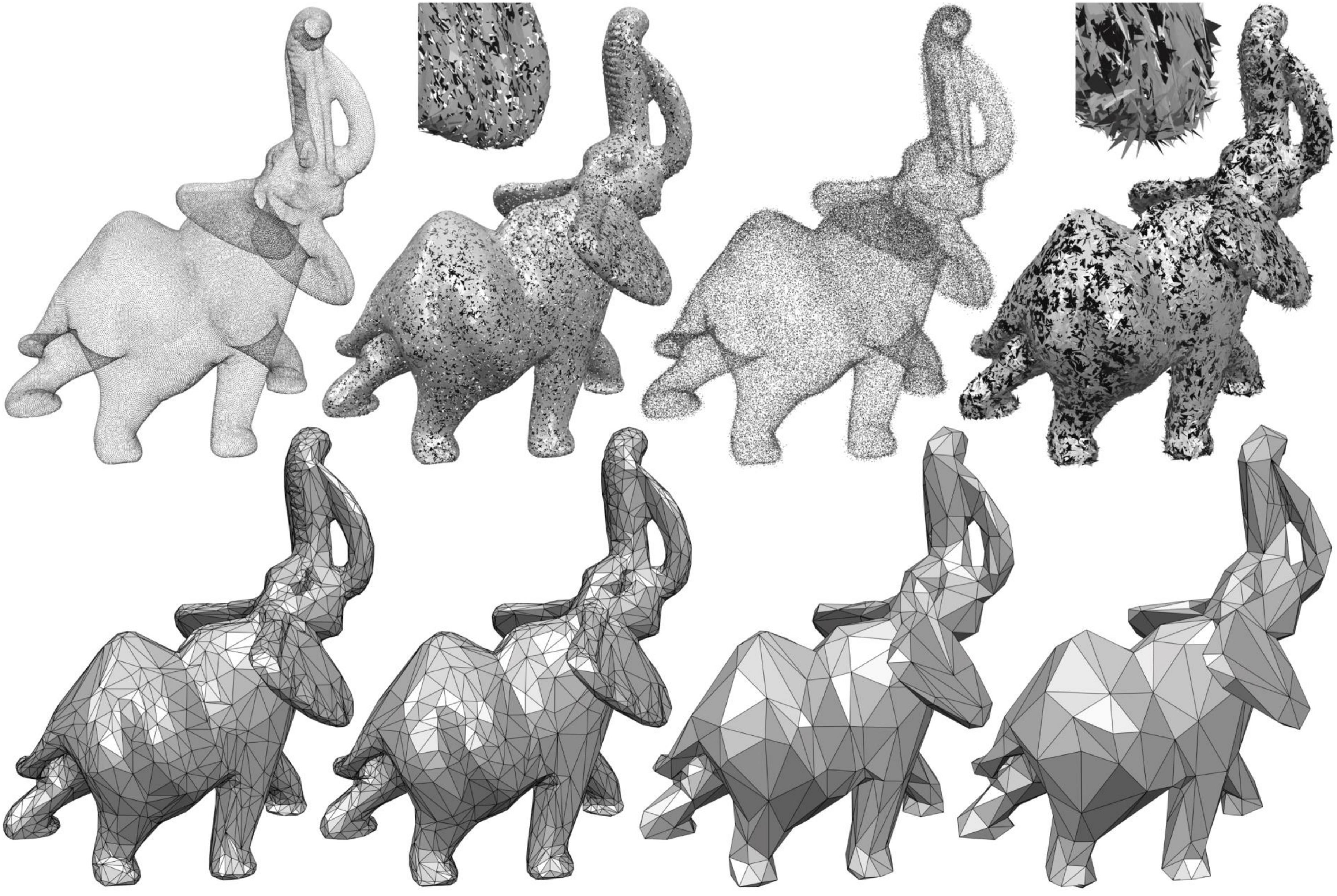


Blade



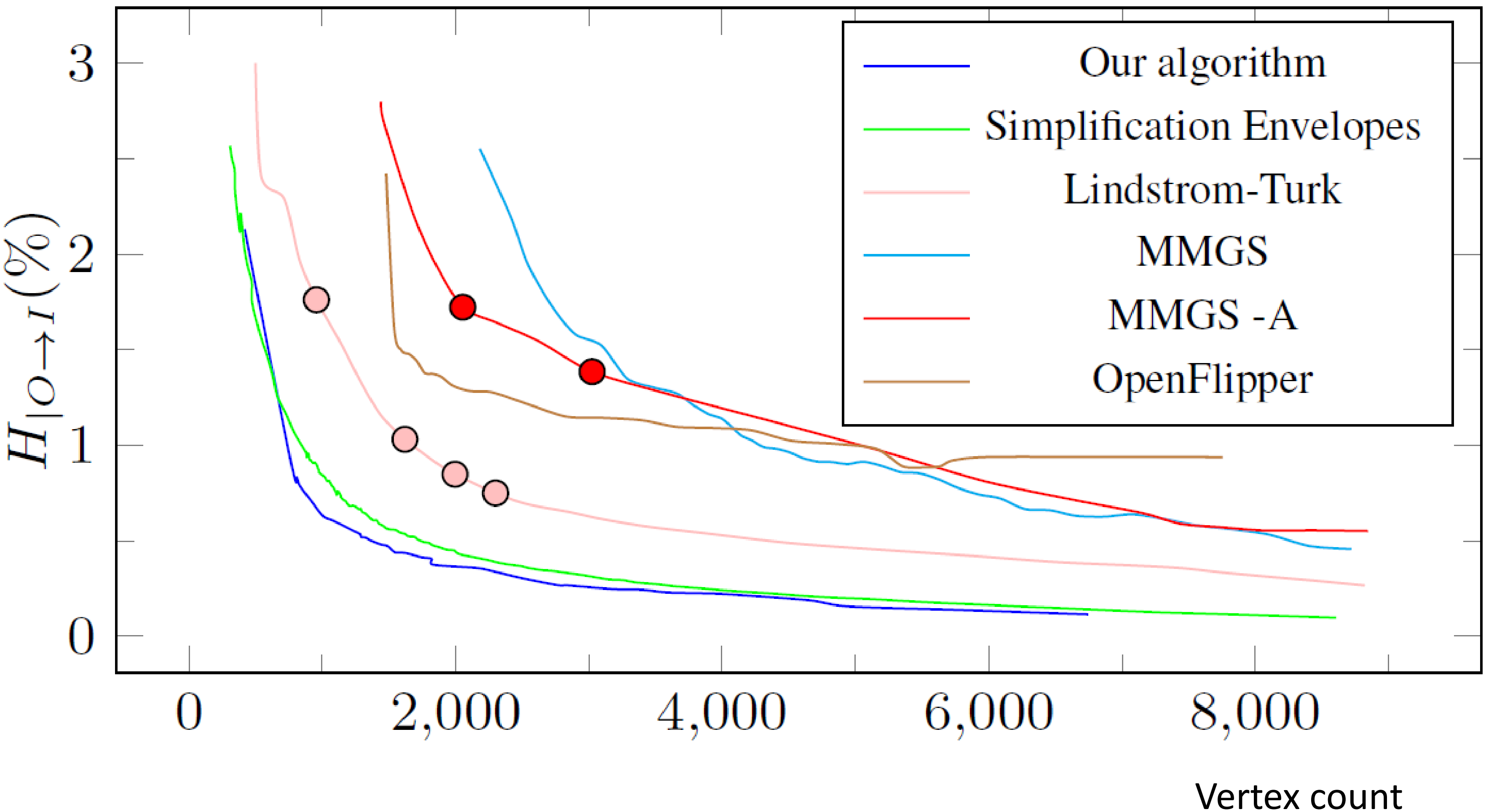
15
10
5
0
0
2
8
10
Vertex count (×10³)
Time (×10³ seconds)
.75
.80
.85
.90
1.0
tolerance

Robustness to defects

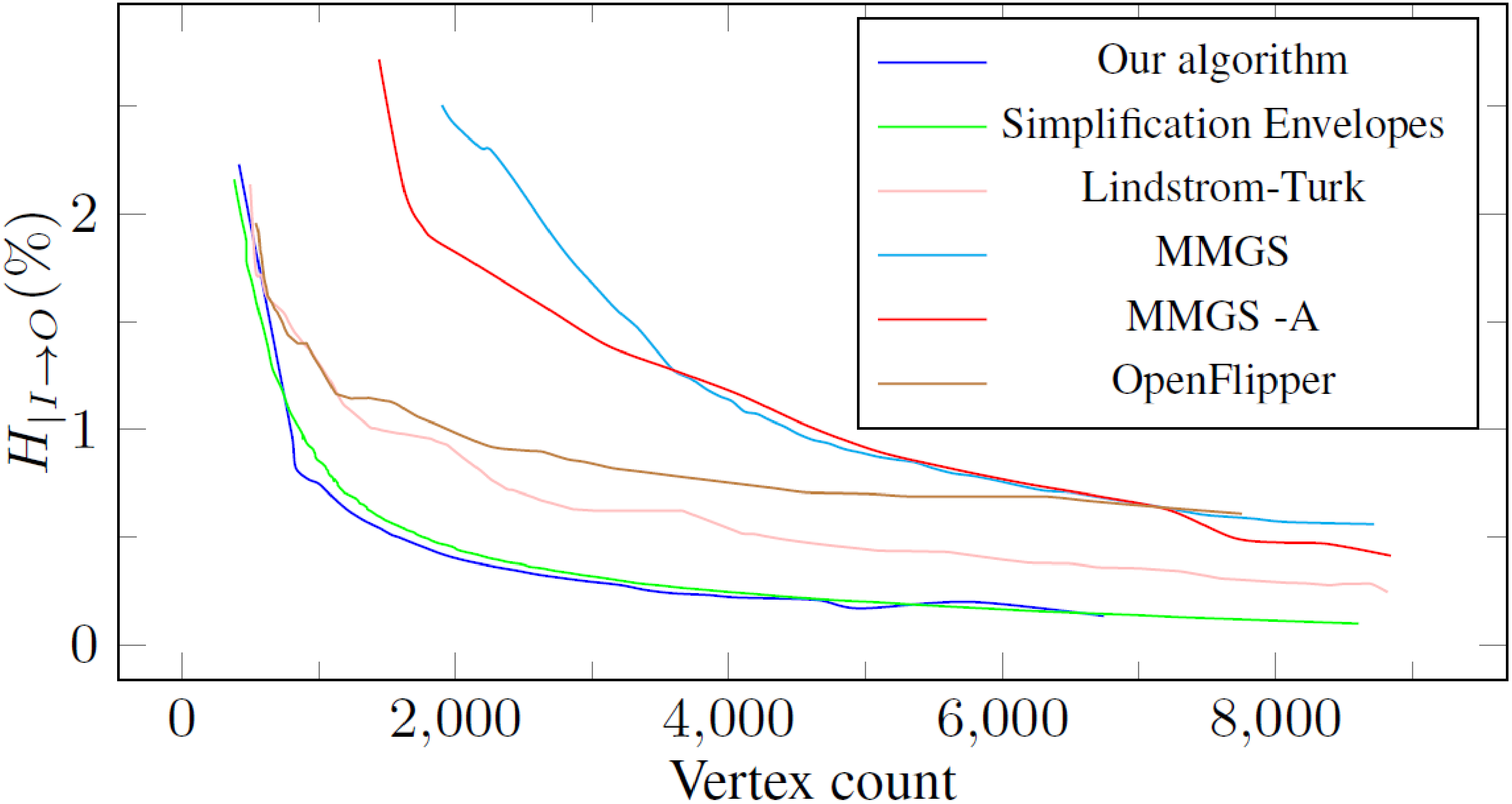


Comparisons

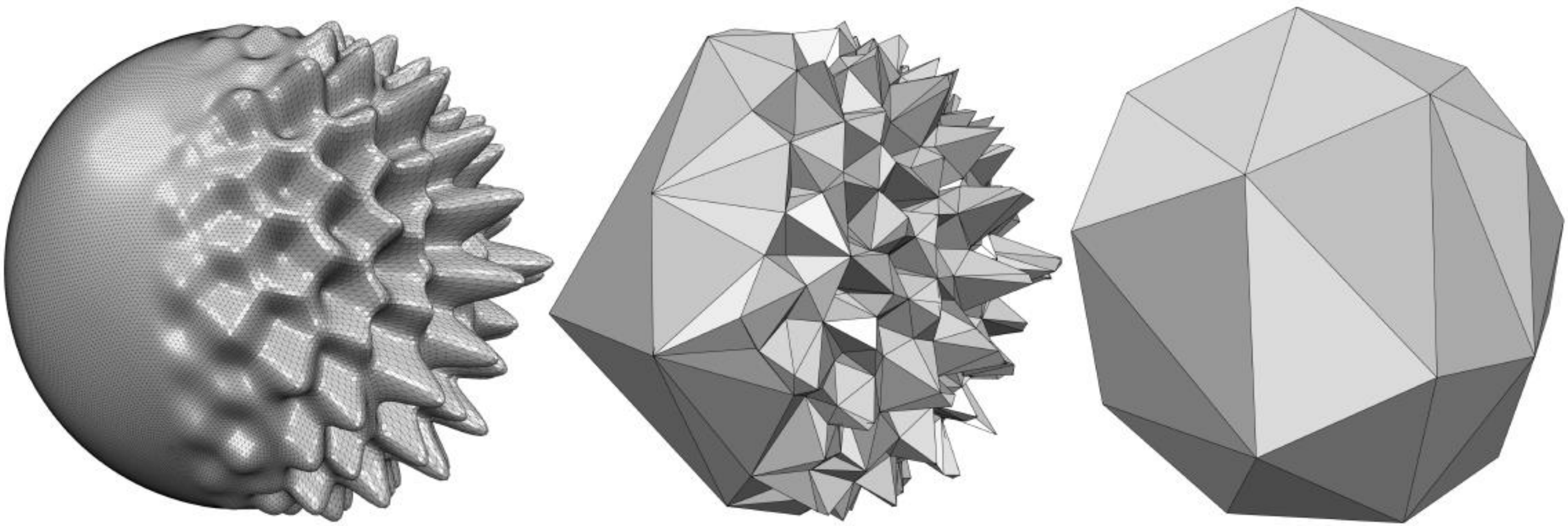
Comparisons – Hausdorff distance (output to input)



Comparisons – Hausdorff distance (output to input)



Simplification Envelopes & Large Tolerance



Input Simplification envelopes Our approach

Large tolerance

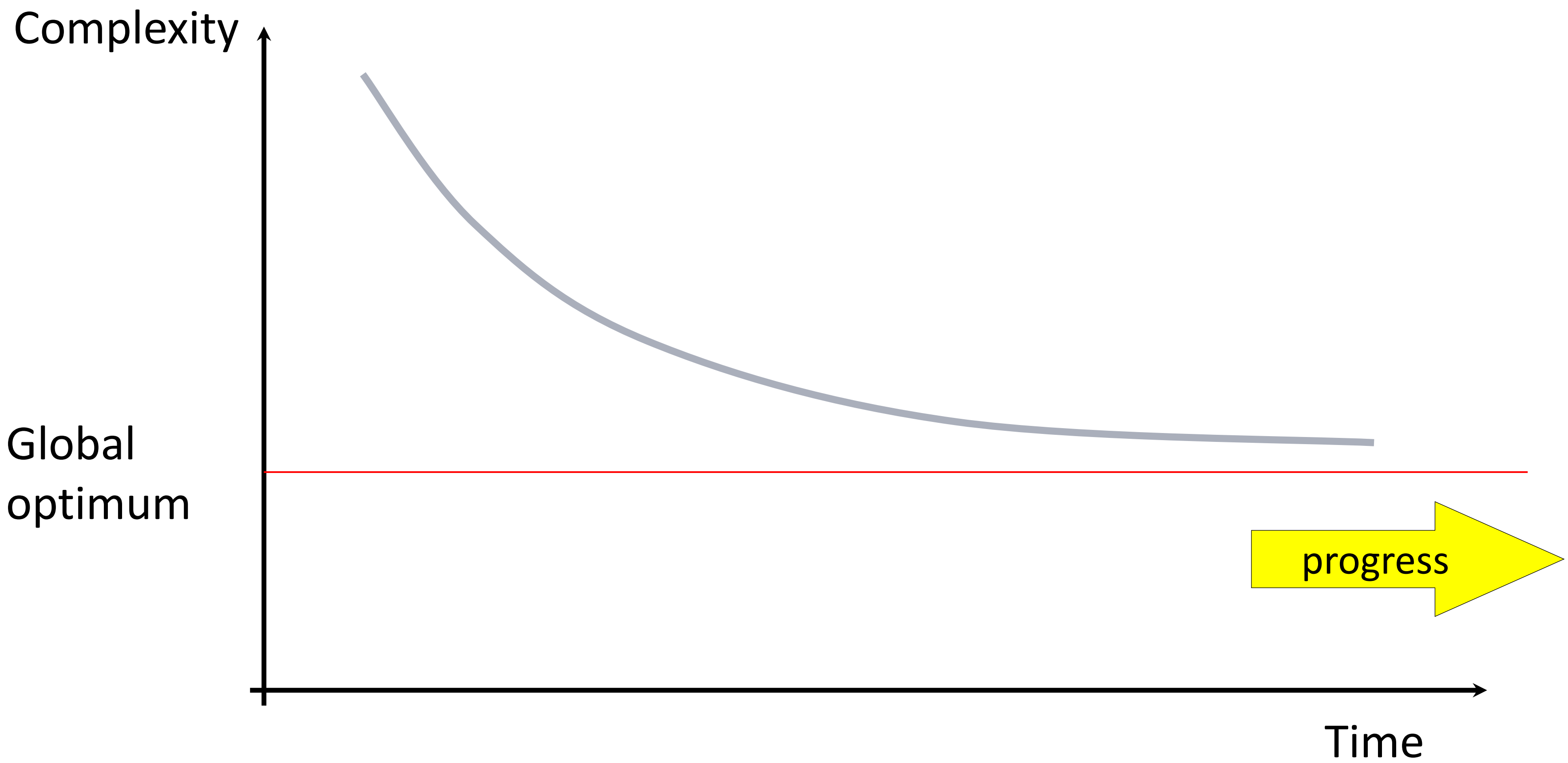
Remaining Challenges

Remaining Challenges

- **Error metric**
  - Sobolev norm (geometry + normals + ...)
  - Two-sided
    - Not symmetrized
    - Mass transport: Wasserstein distance [Mérigot, Peyré, Solomon]
  - Robustness to wide range of defects
    - Mass transport [Digne, Cohen-Steiner, Desbrun, A.]
  - Structure-aware

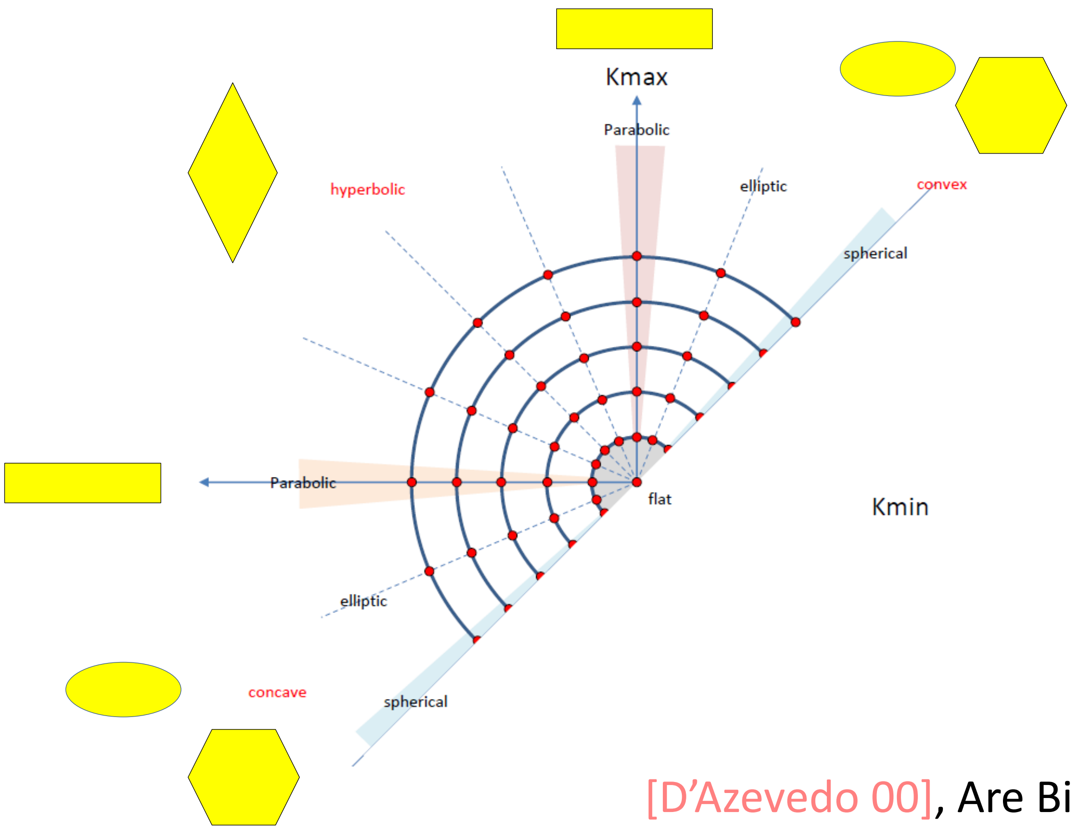
Remaining Challenges

- **Progressive algorithm**



Remaining Challenges

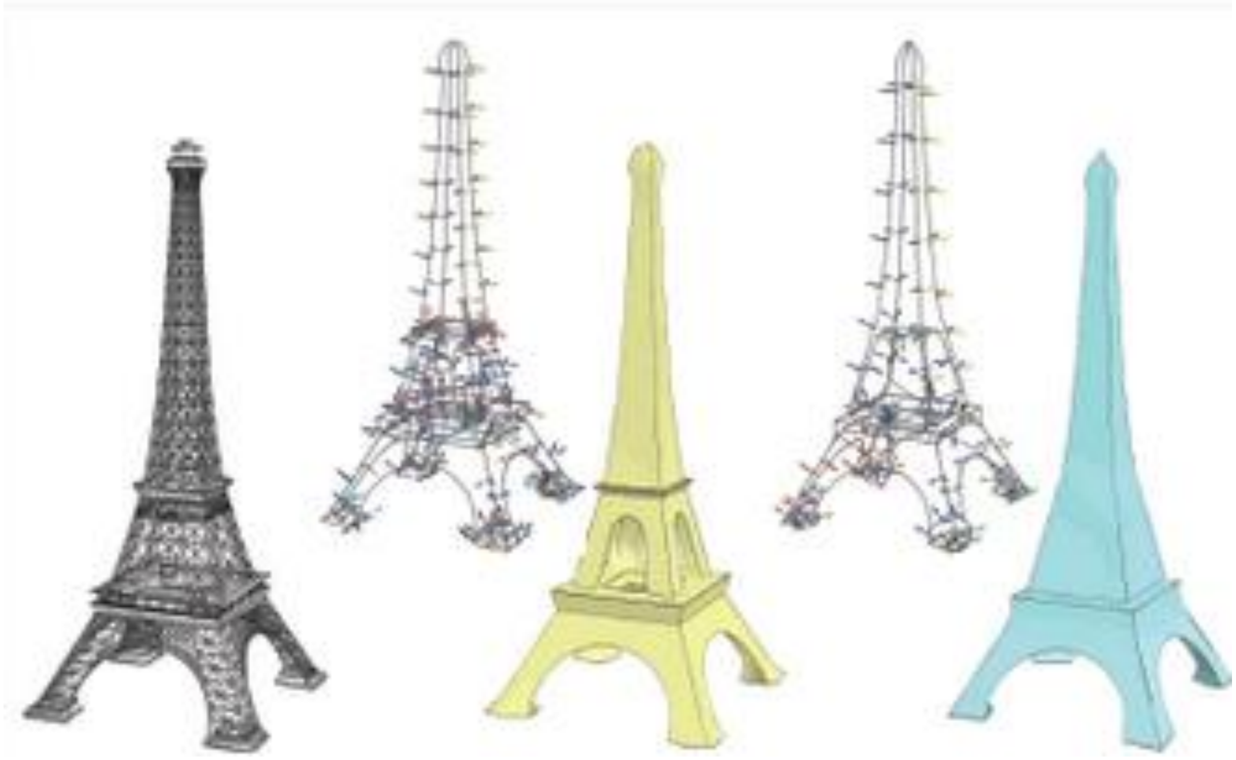
- **Polygons?**



[D'Azevedo 00], Are Bilinear Quadrilaterals Better Than Linear Triangles?

Remaining Challenges

- **Beyond approximation**
  - Abstraction [Sheffer, Mitra et al. 2009] *Abstraction of Man-Made Shapes*.



Remaining Challenges

- **Beyond approximation**
  - *Meaningful* LODs. [Verdié, Lafarge, A. 2013]