International Geometry Summit 2016
SGP | SMI | SPM
Berlin, June 20–24th

QUAD MESHING

The Problem of Quadrilateral Surface Mesh Generation from a Computer Graphics & Geometry Processing Perspective

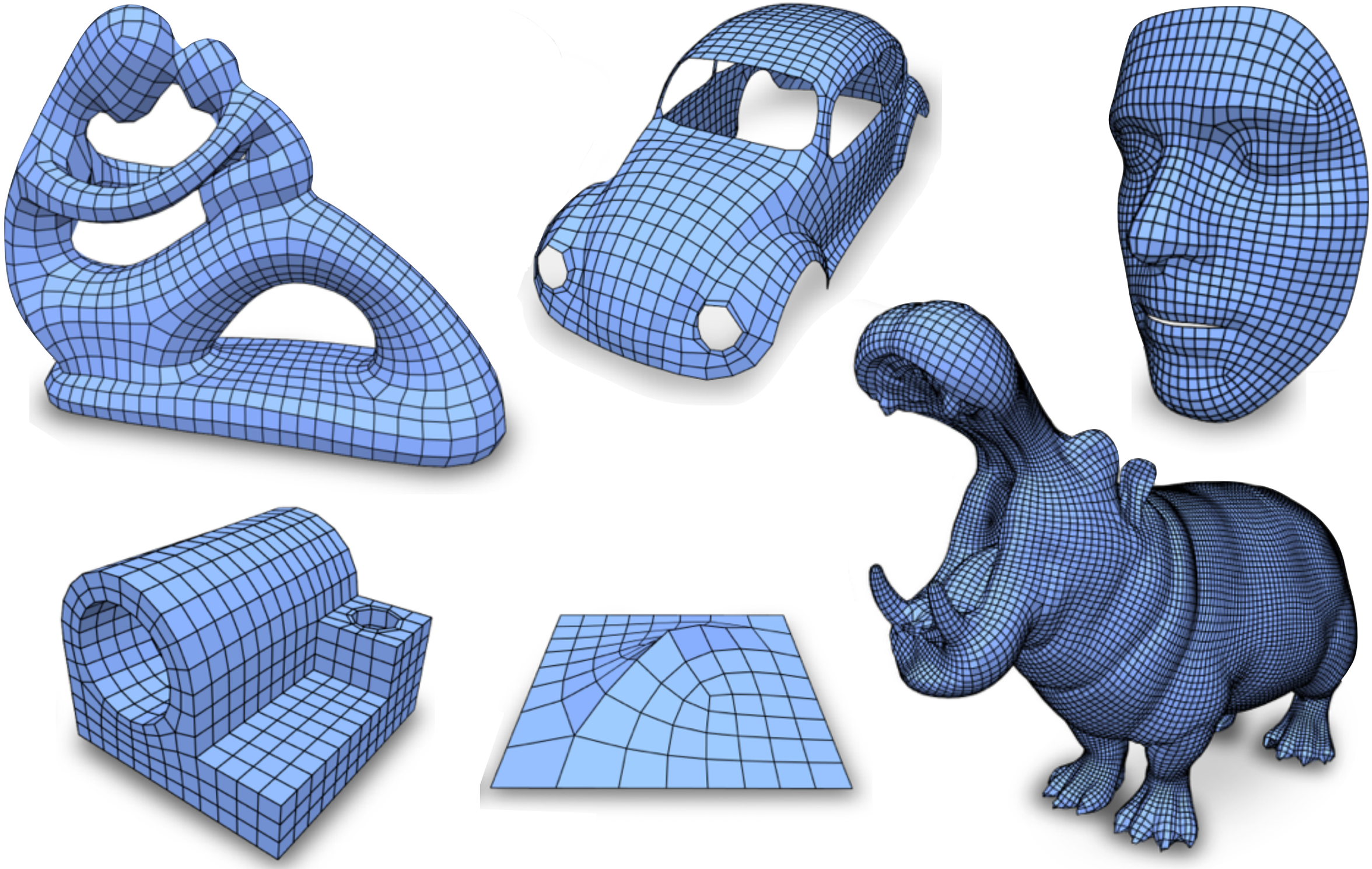
Marcel Campen

New York University

NYU

SGP Summer School 2016

QUAD MESHES?



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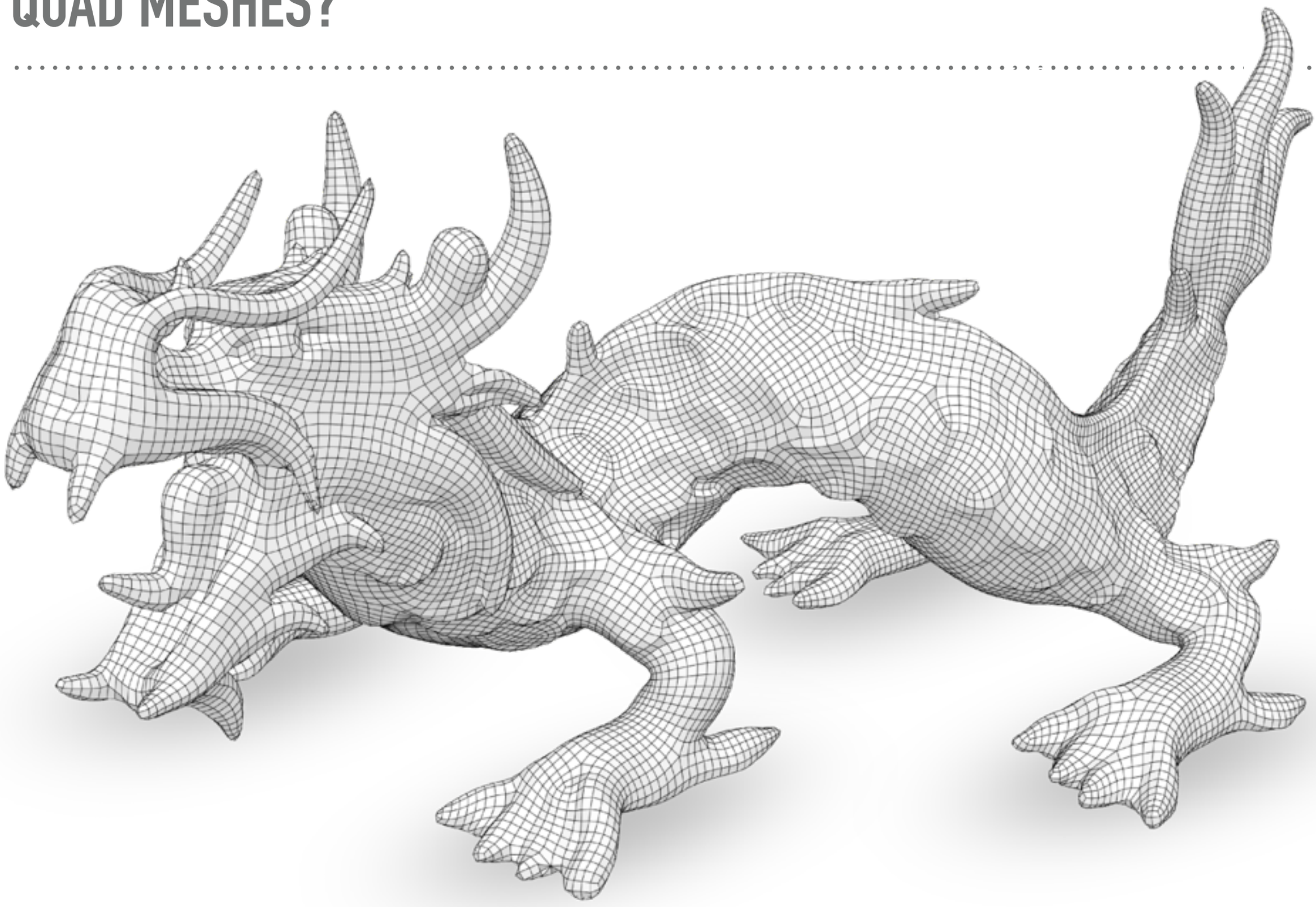

WHY QUAD MESHES?

- alignment to principal curvature directions
- tensor-product structure (for NURBS, etc.)
- anisotropy without bad angles
- bilinear interpolation
- higher efficiency in numerical simulations
- aesthetics
- legacy reasons

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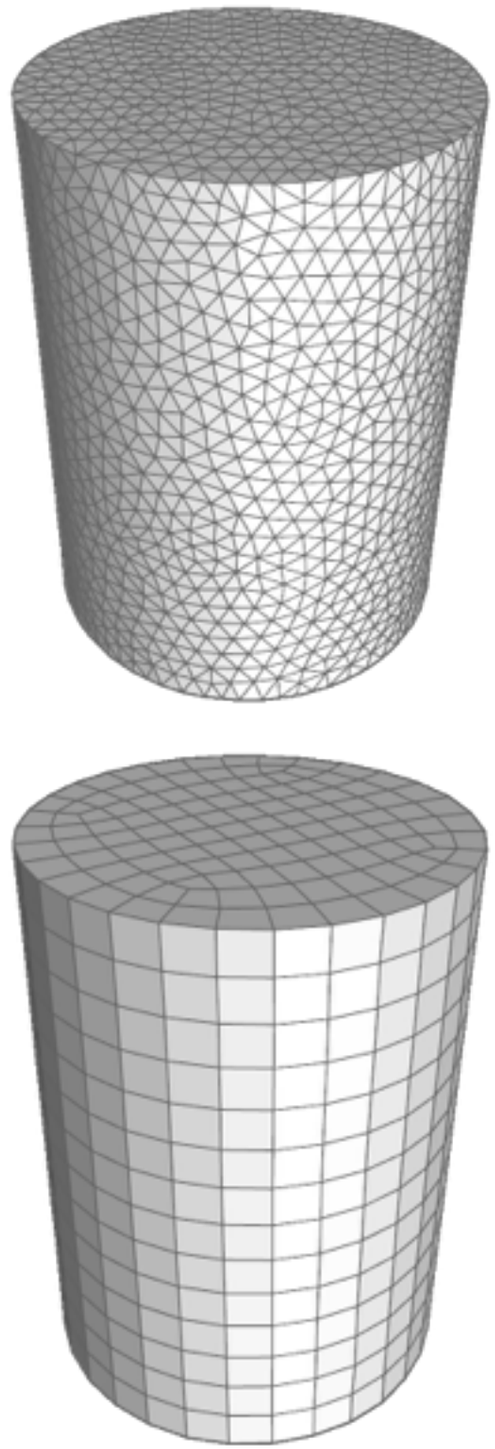

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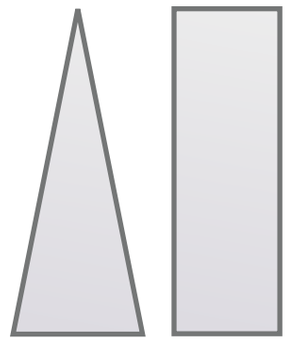

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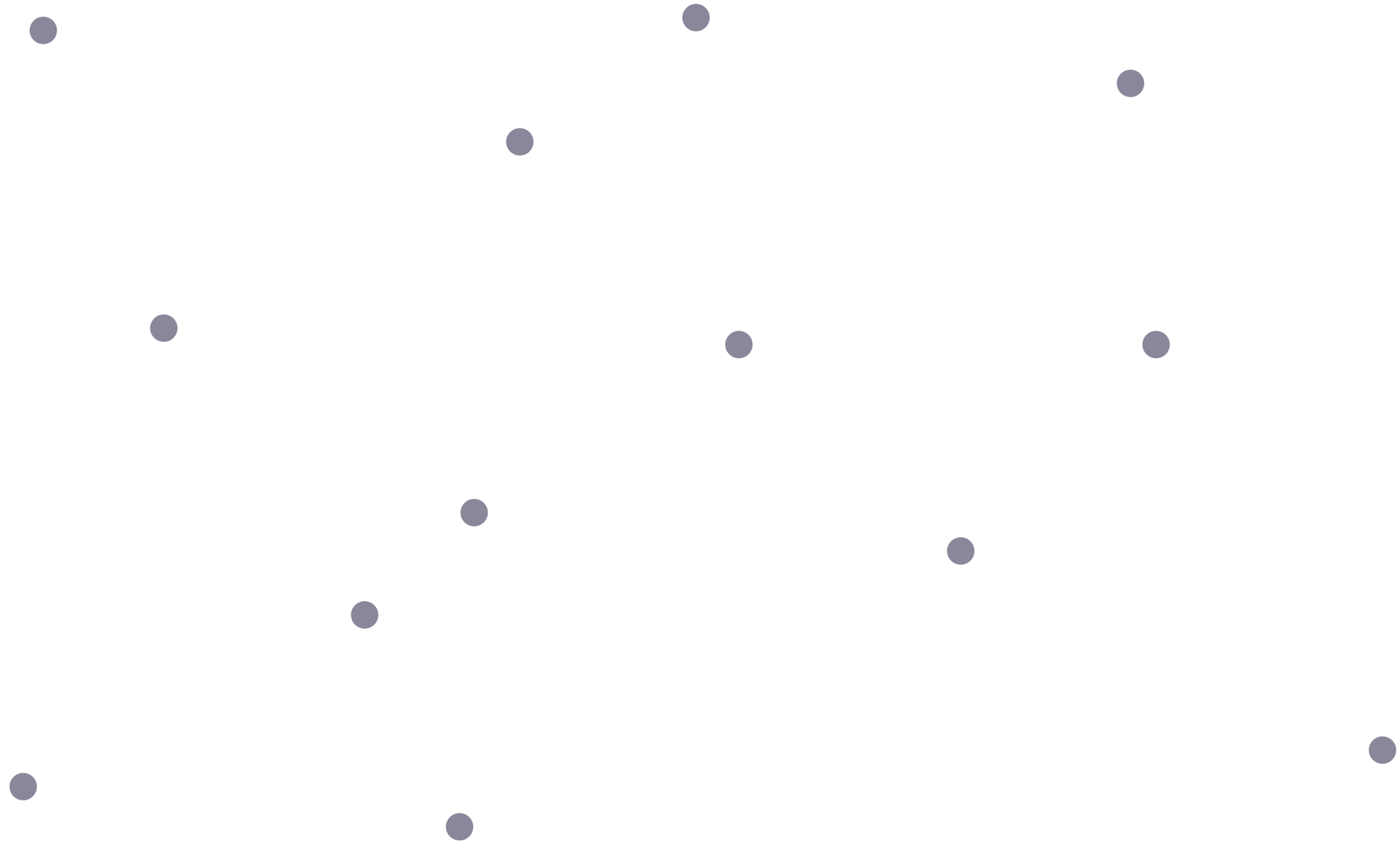

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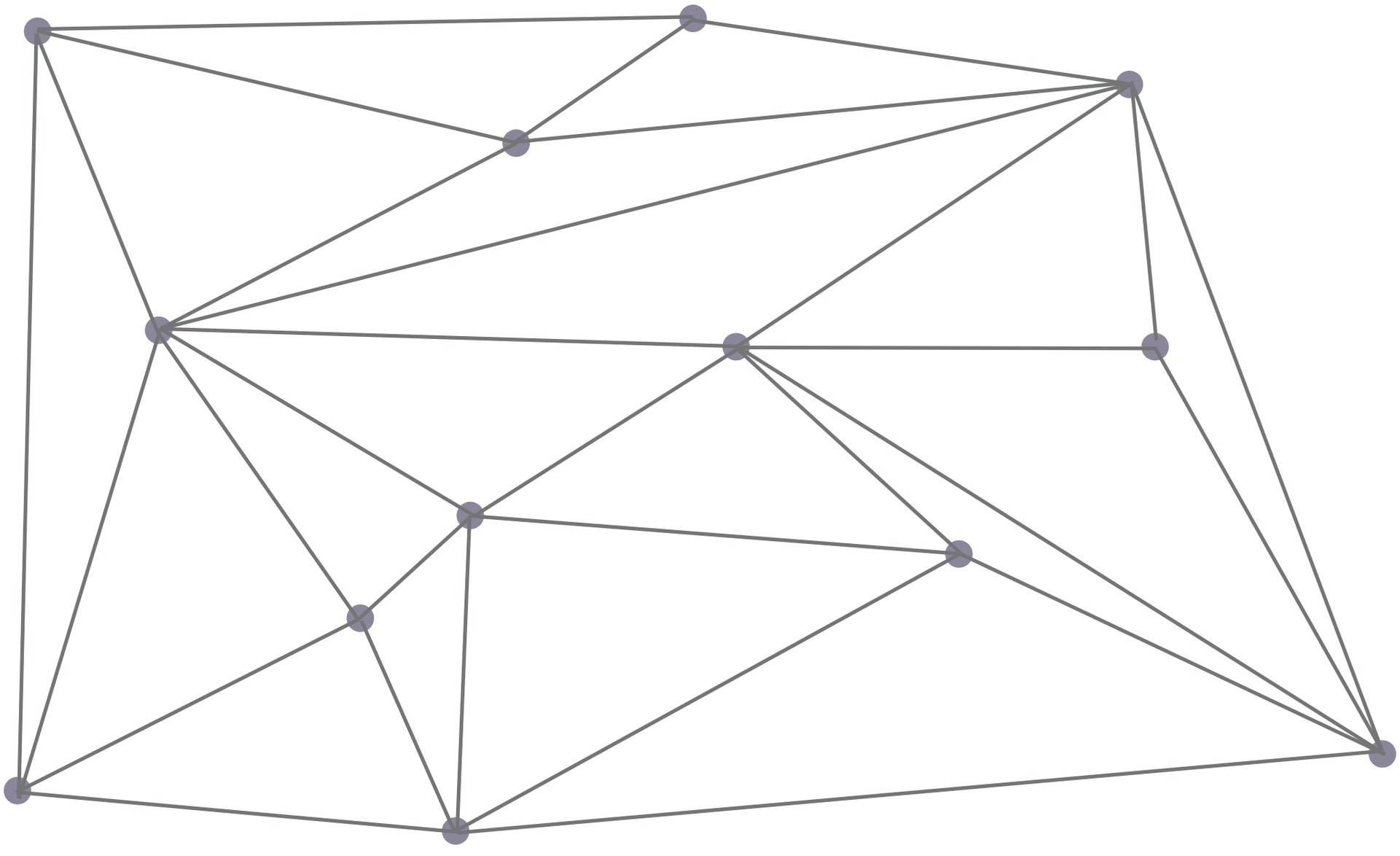
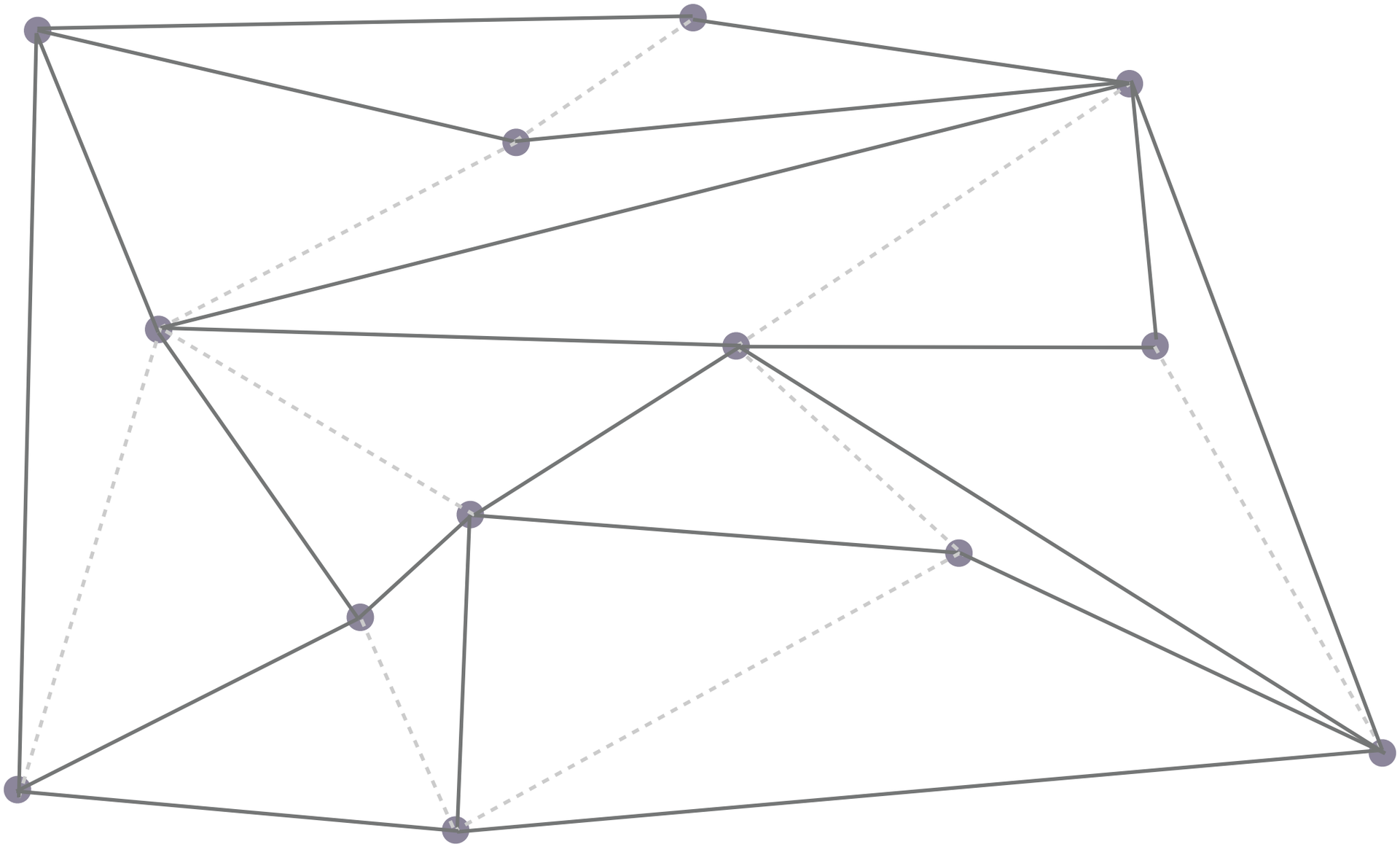


TRIANGLE PAIRING



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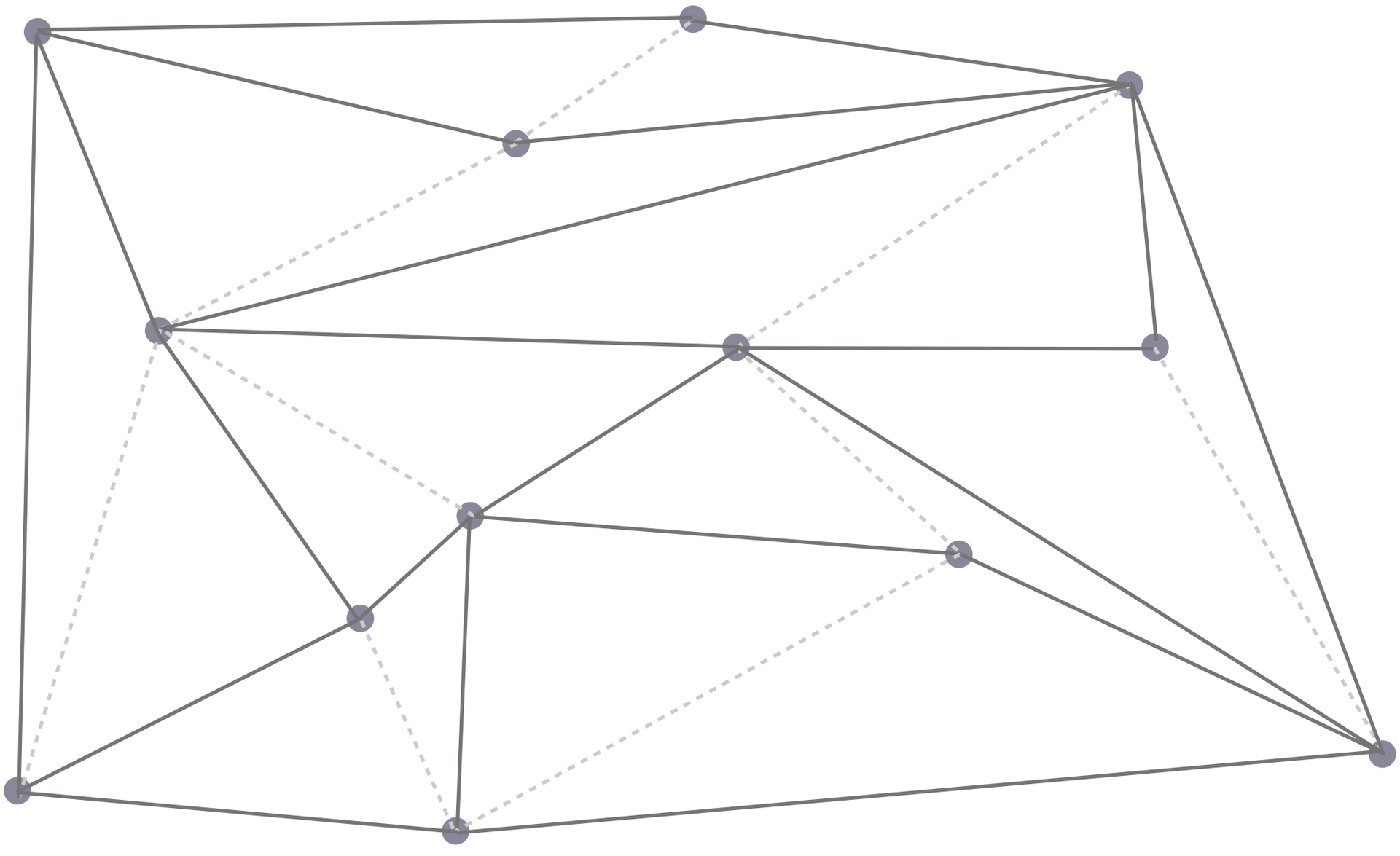

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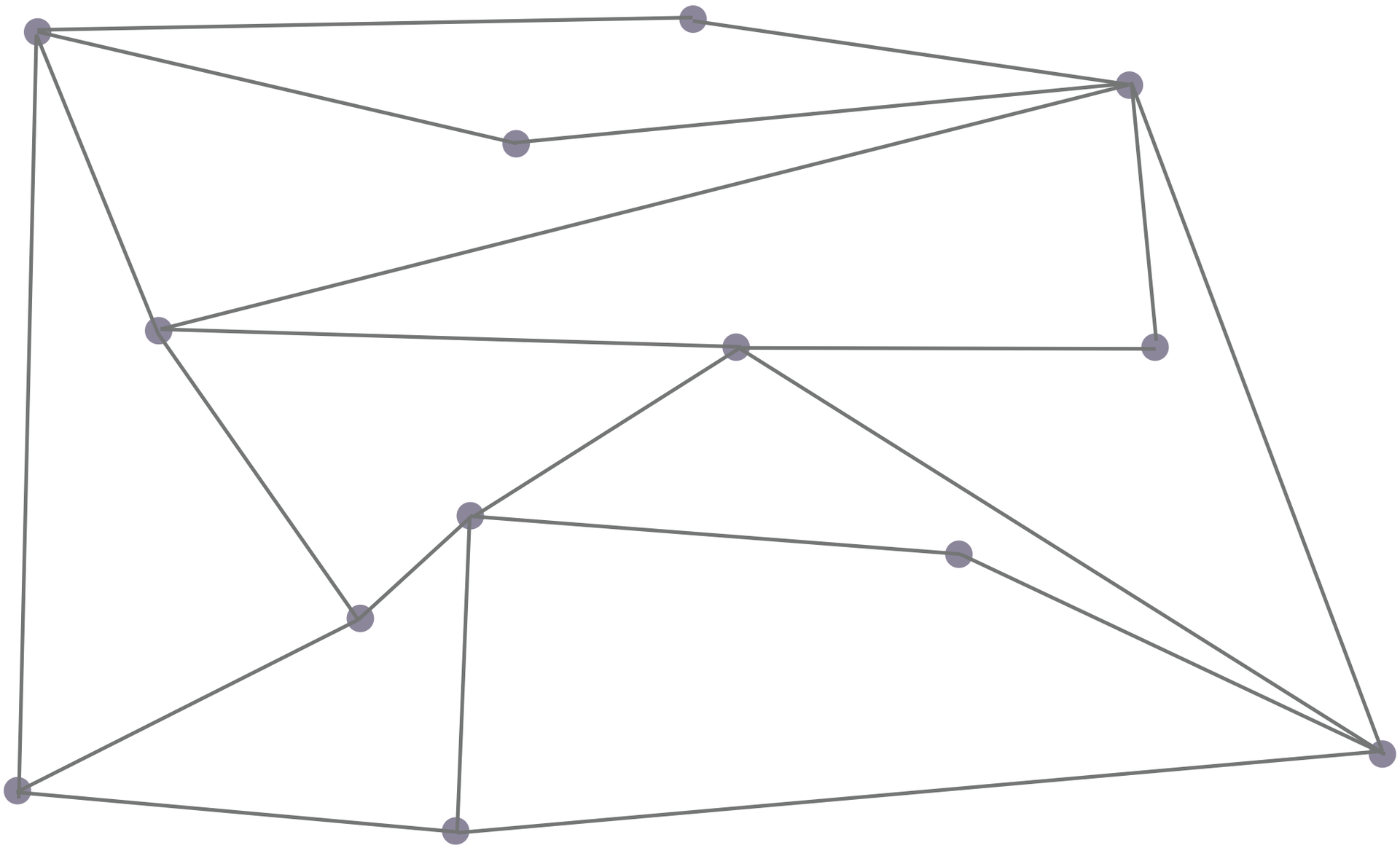
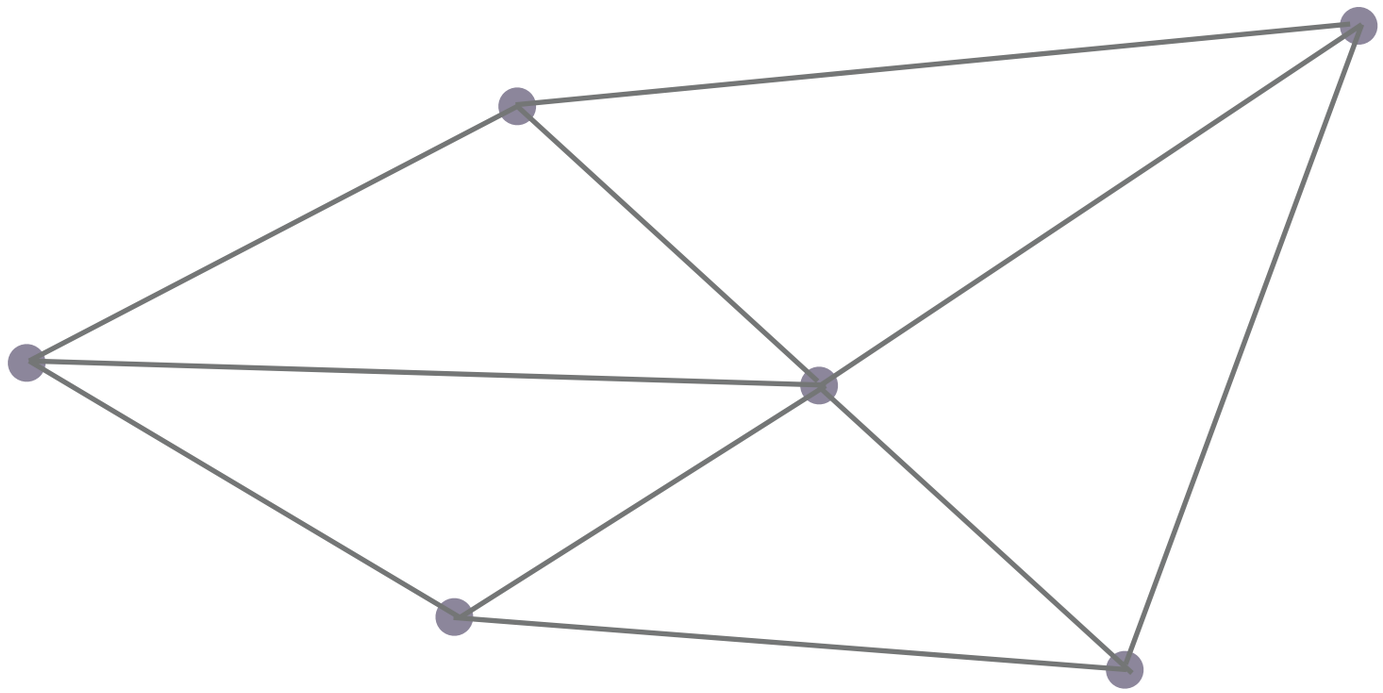


CATMULL-CLARK SPLIT

- Barycentric subdivision of the entire mesh



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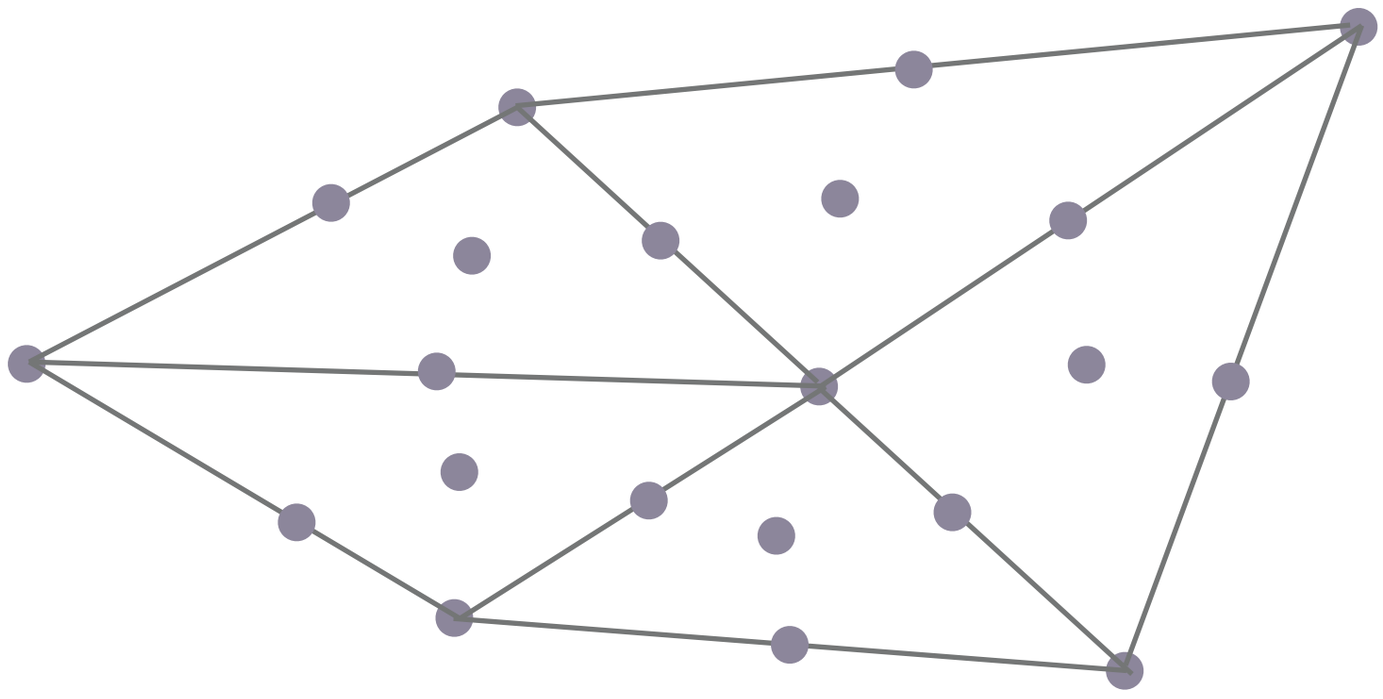

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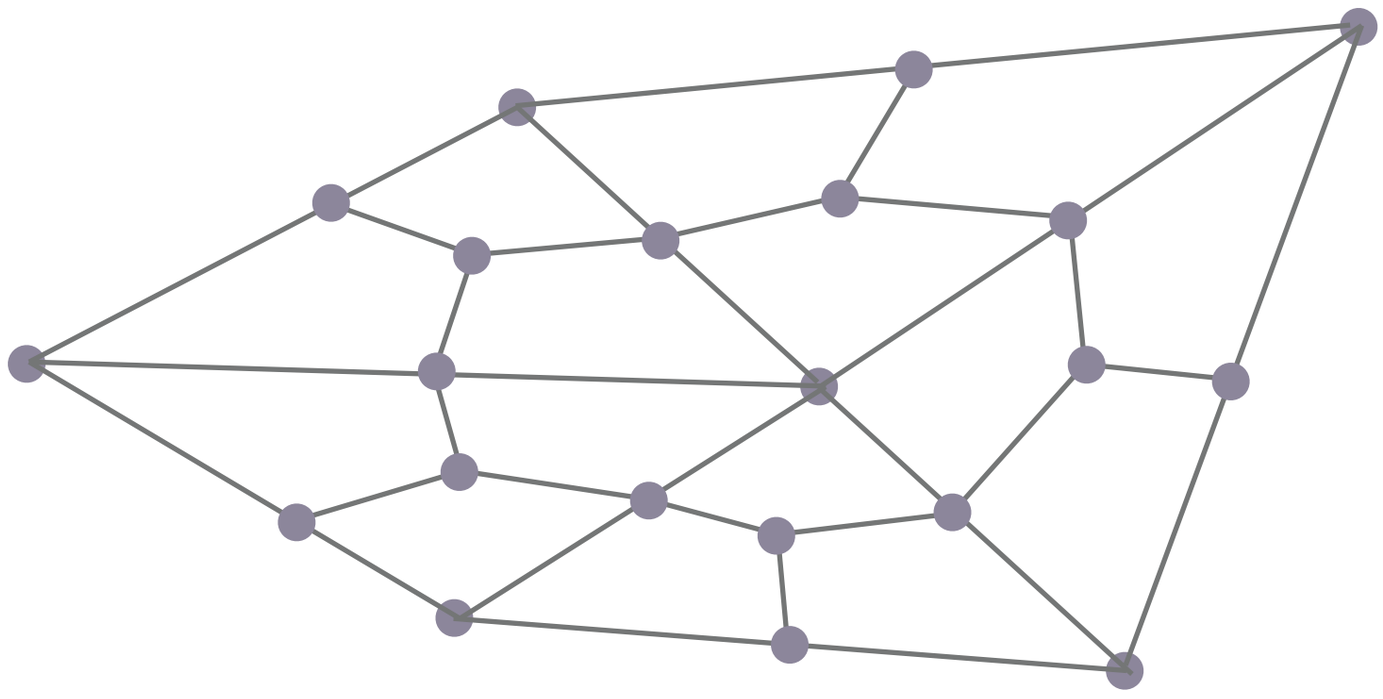
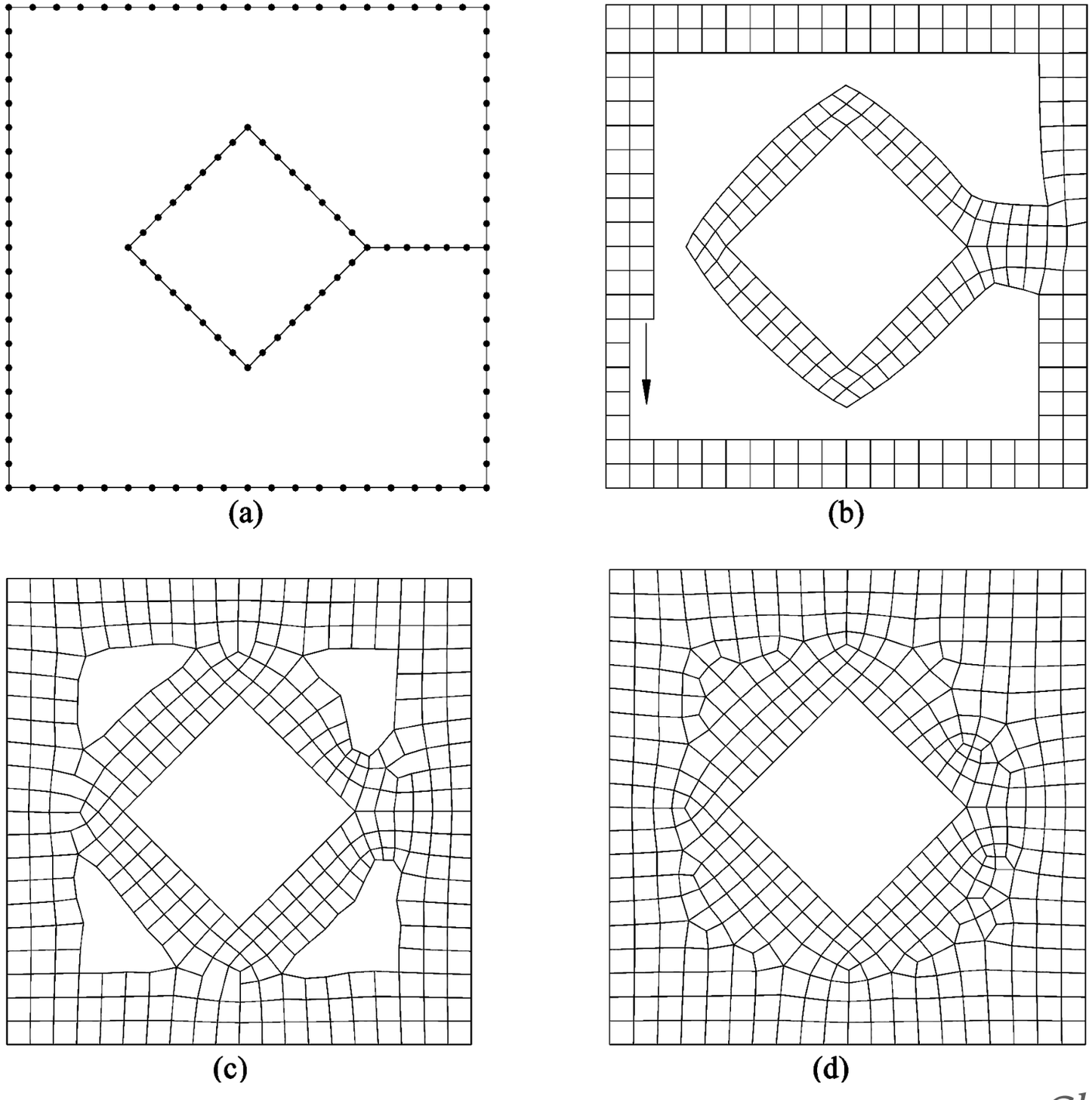


ENGINEERING APPROACHES

- Paving



Choi and Kim 2011

TRACING

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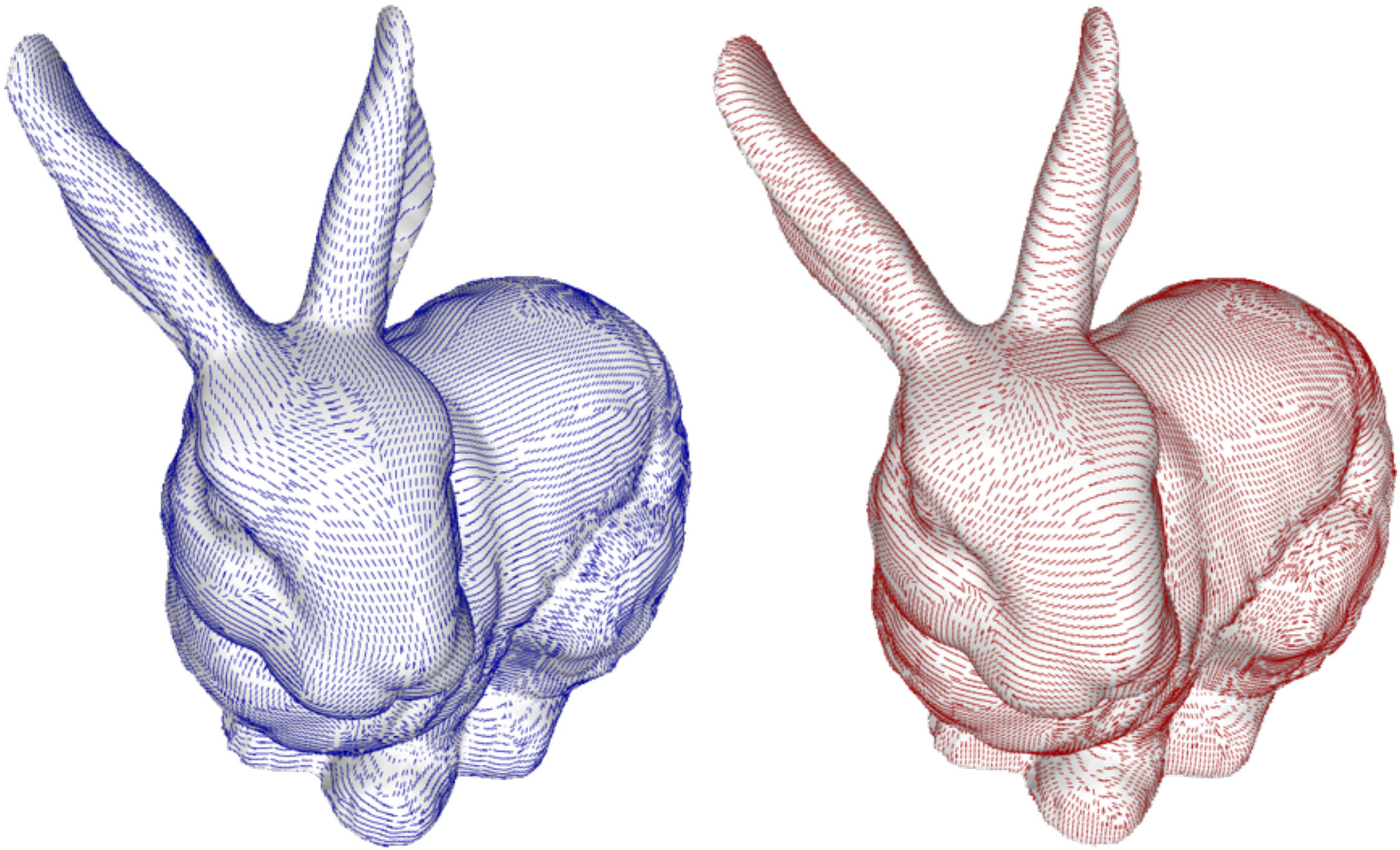

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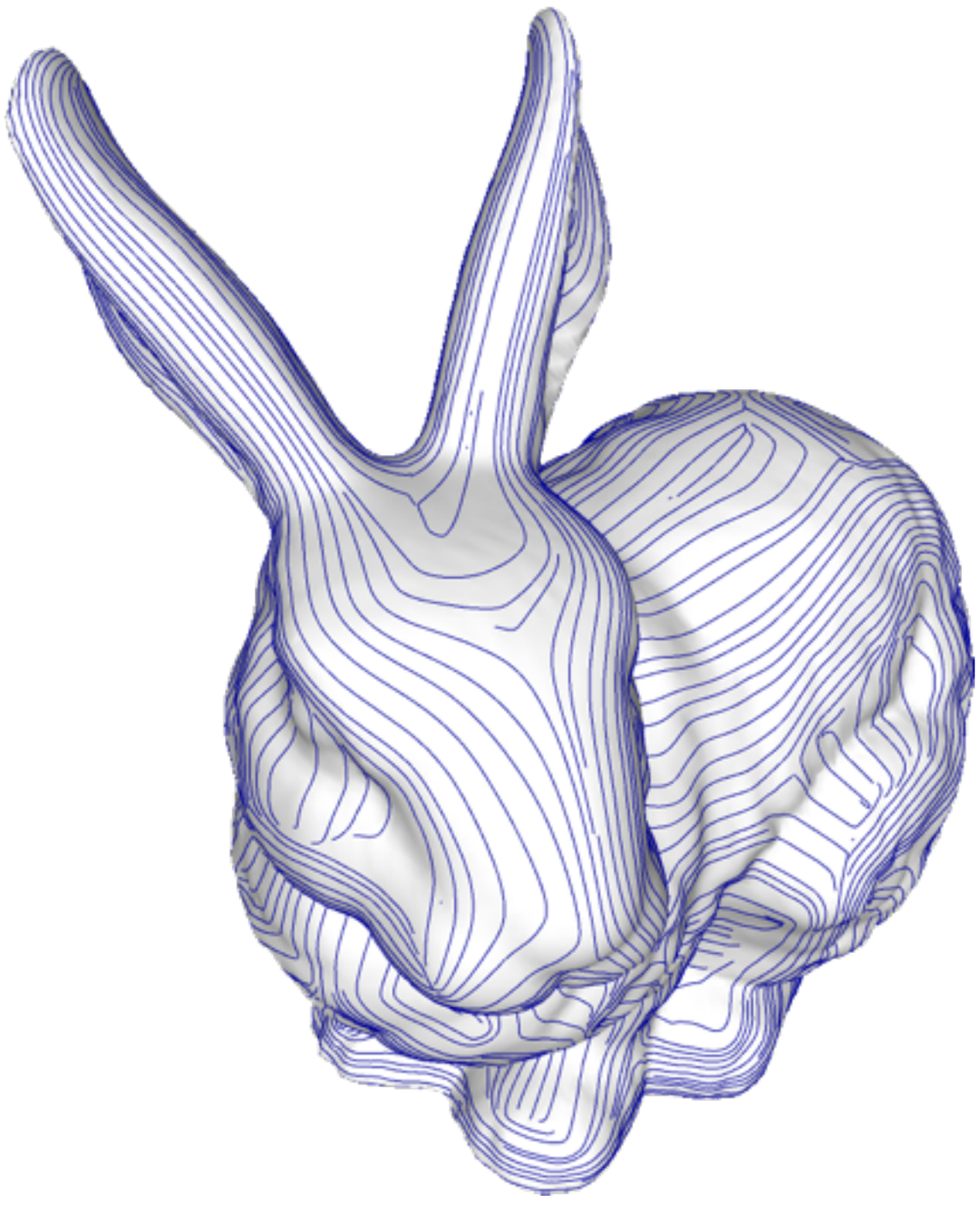

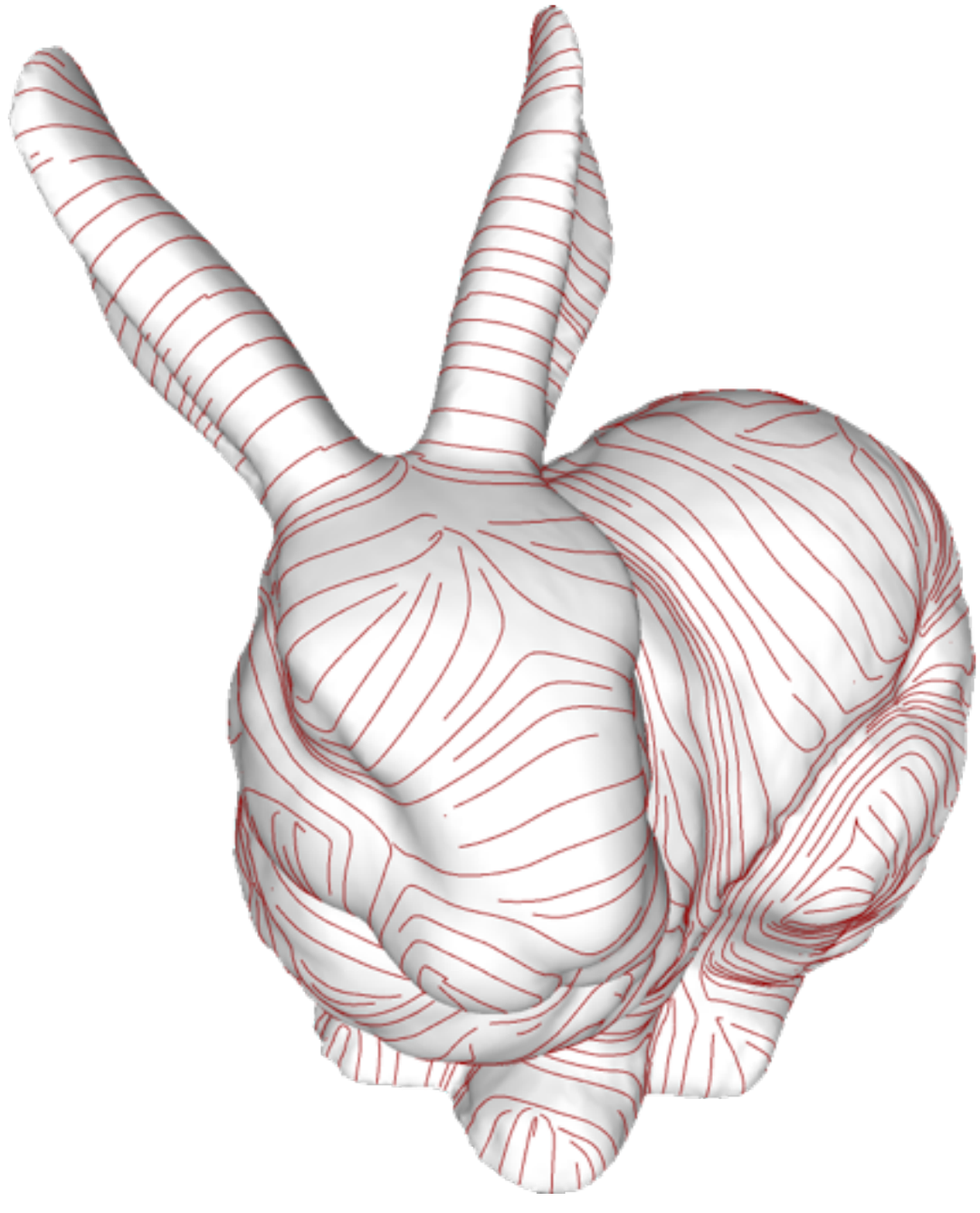

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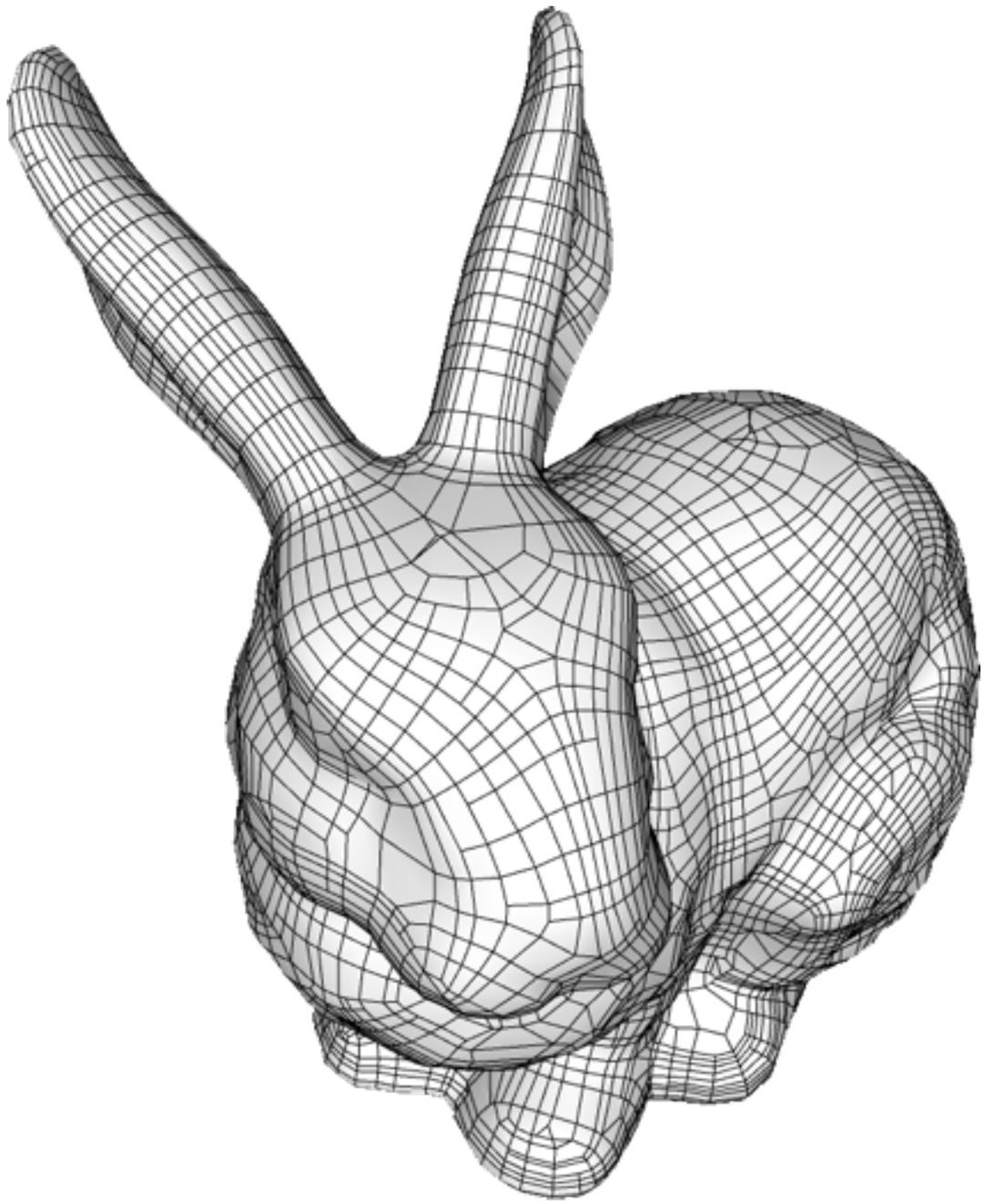

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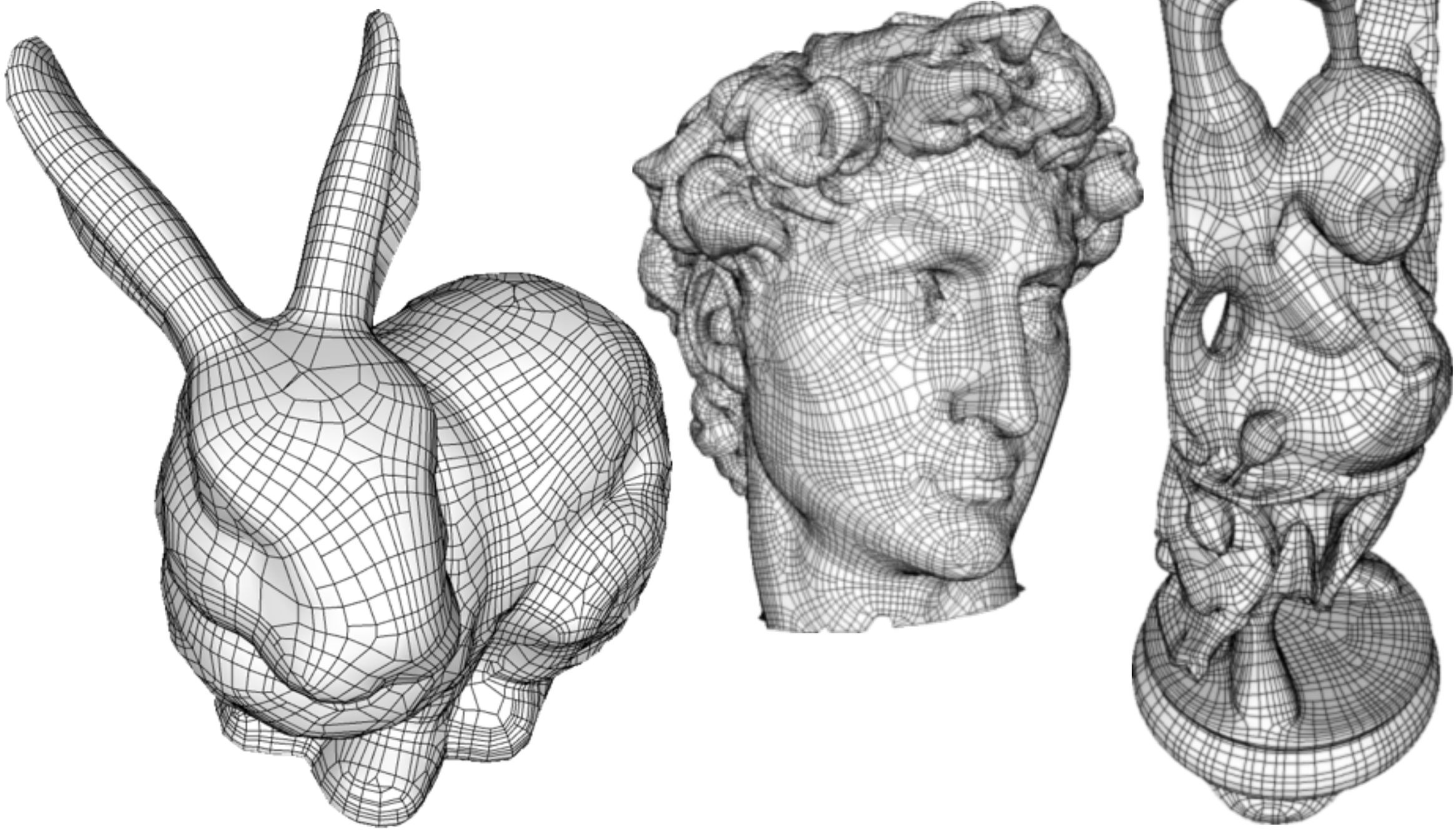
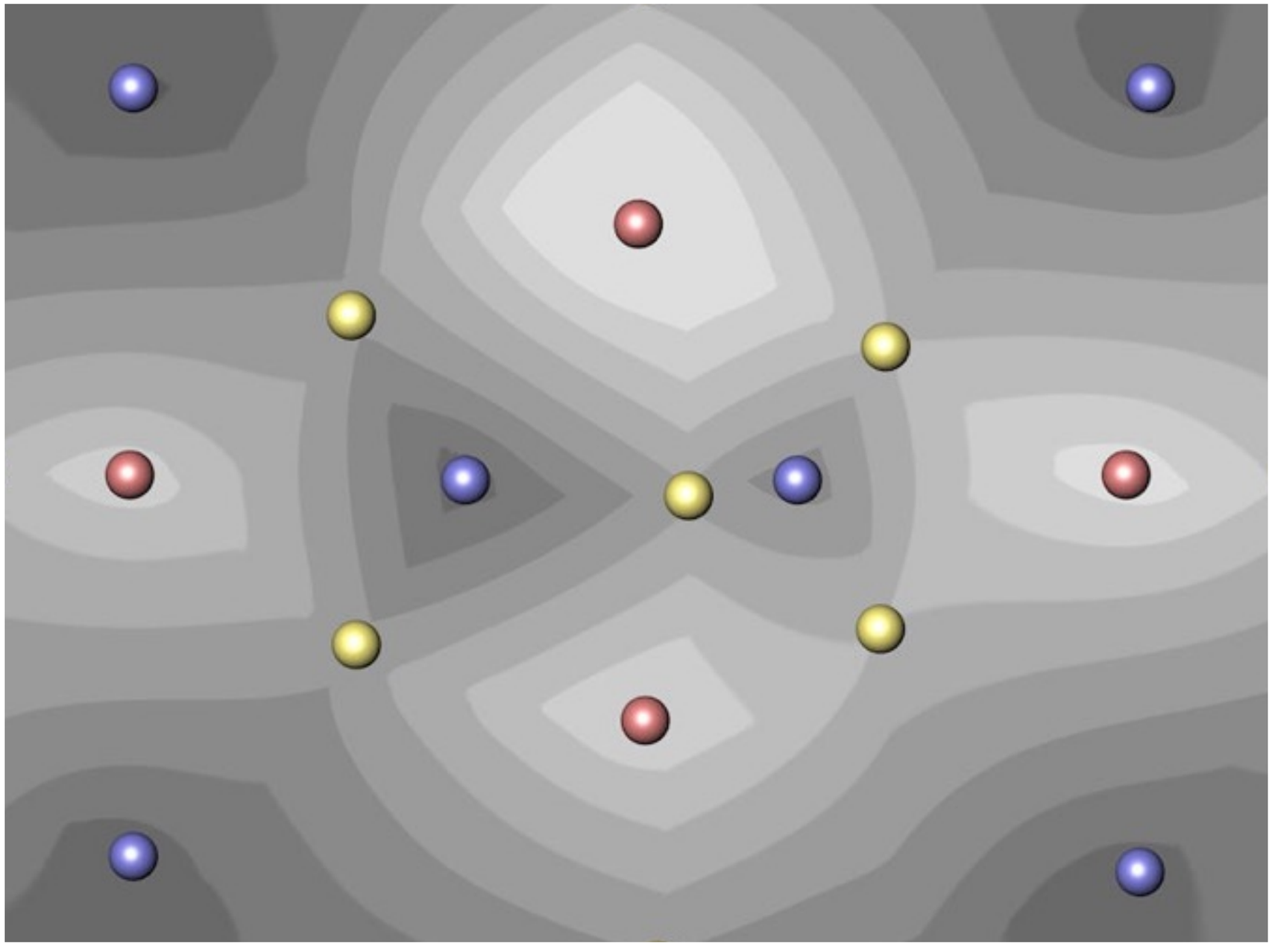


MORSE-SMALE



Weinkauf et al. 2010

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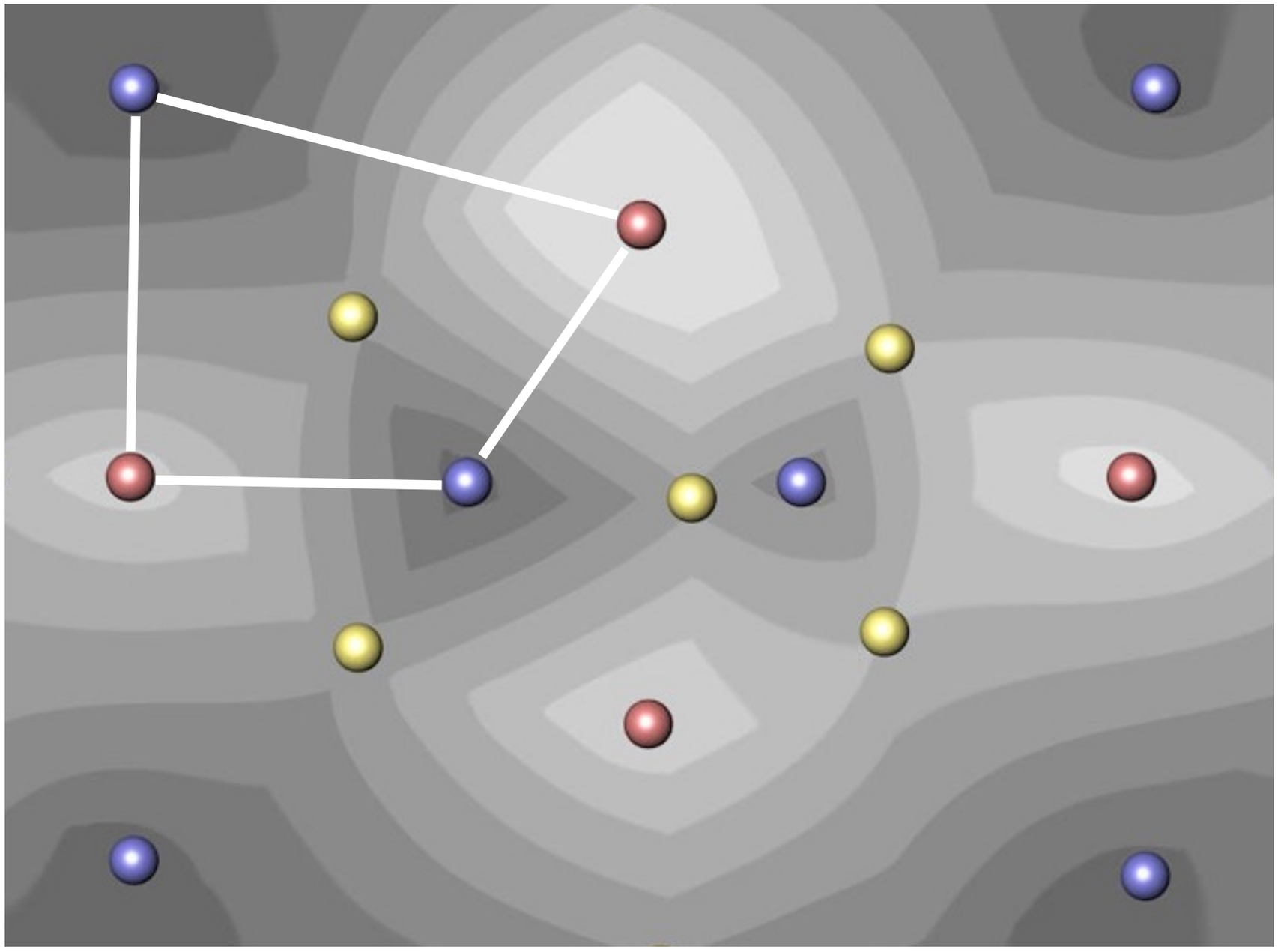

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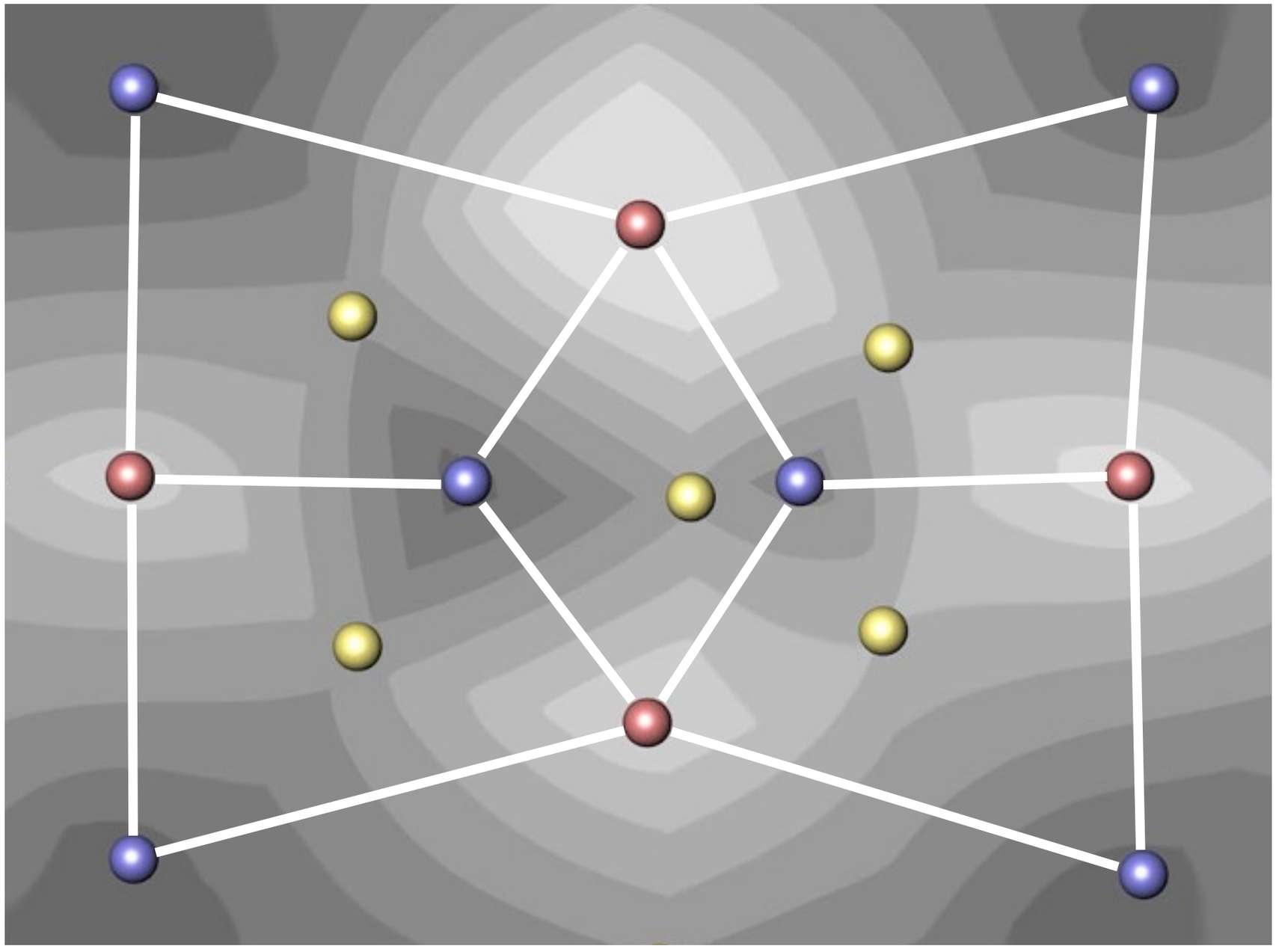

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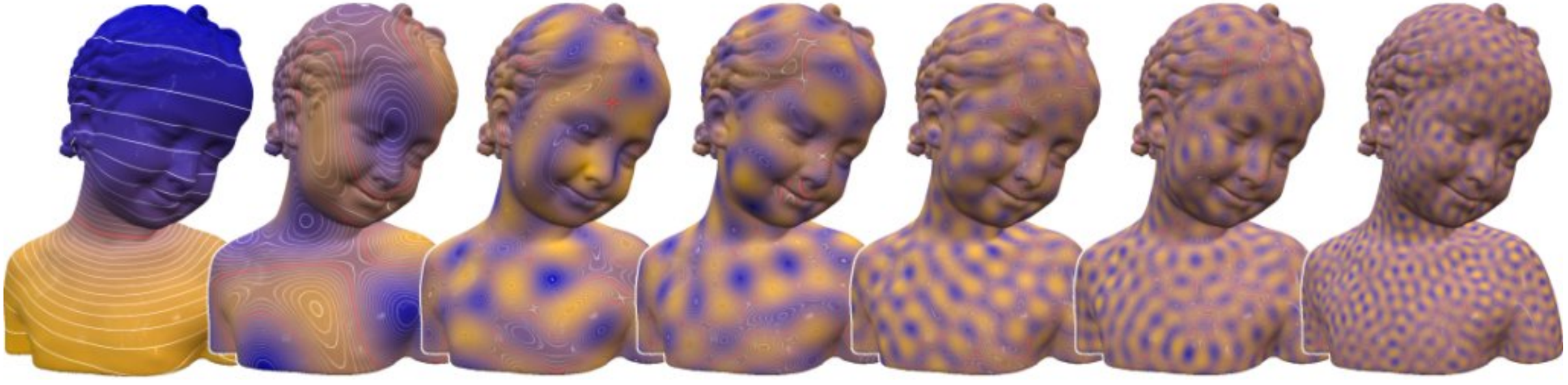
- Eigenfunctions of the Laplace-Beltrami operator



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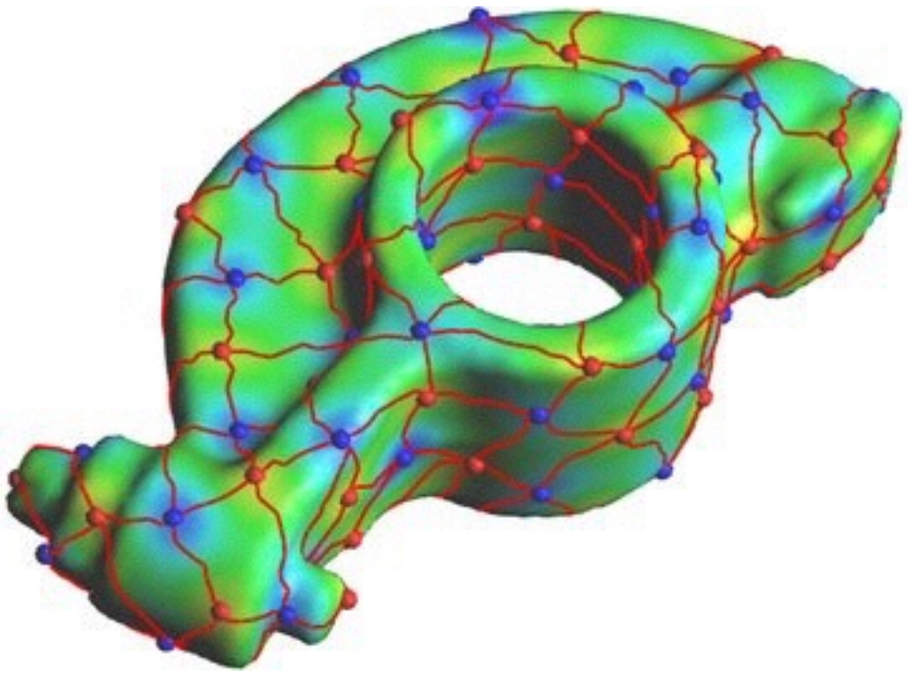

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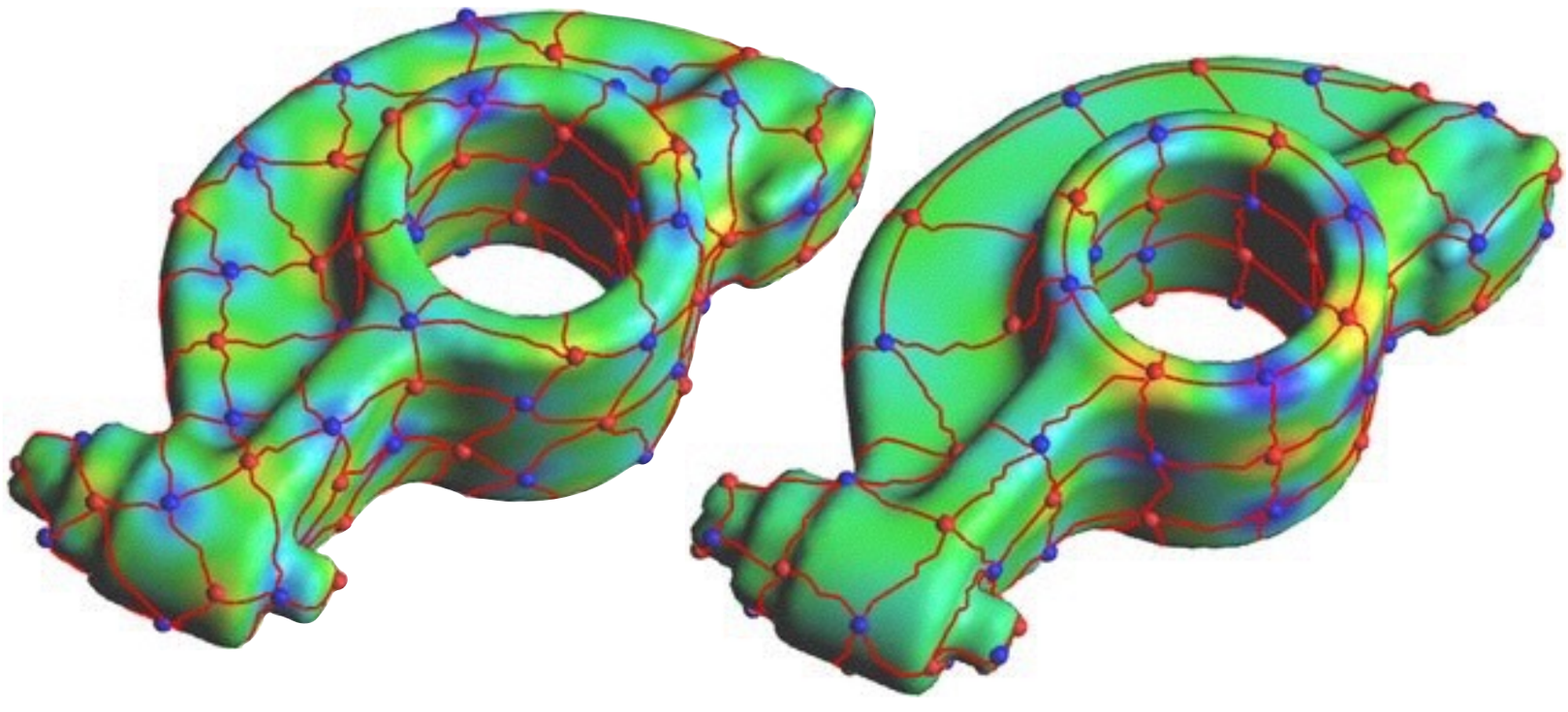

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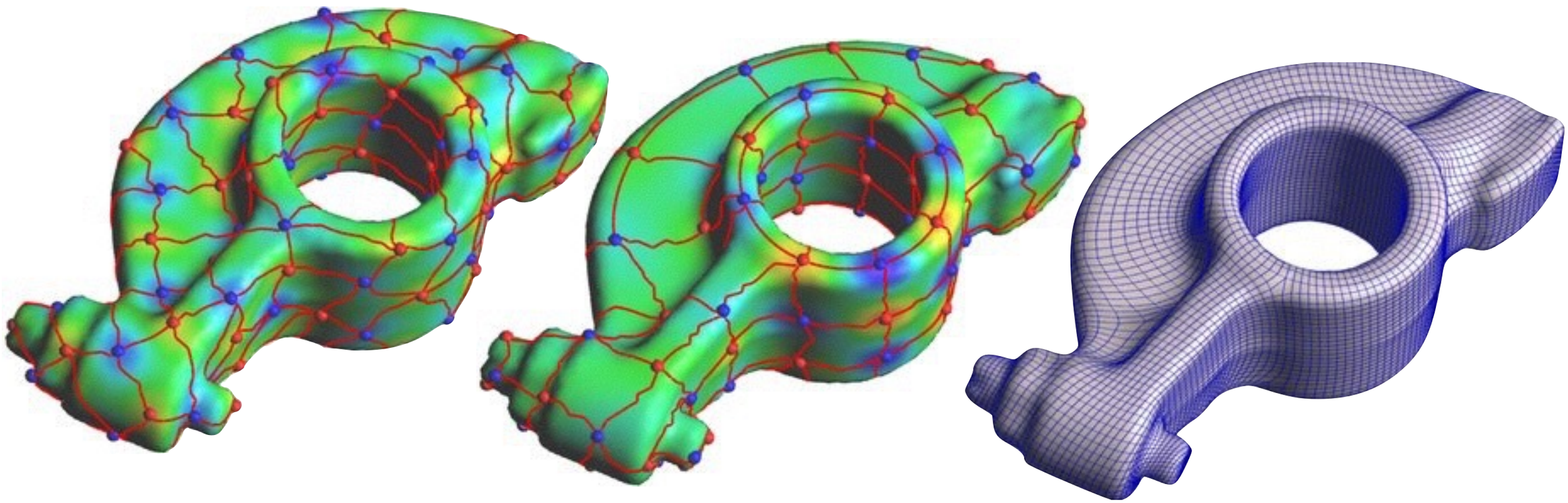

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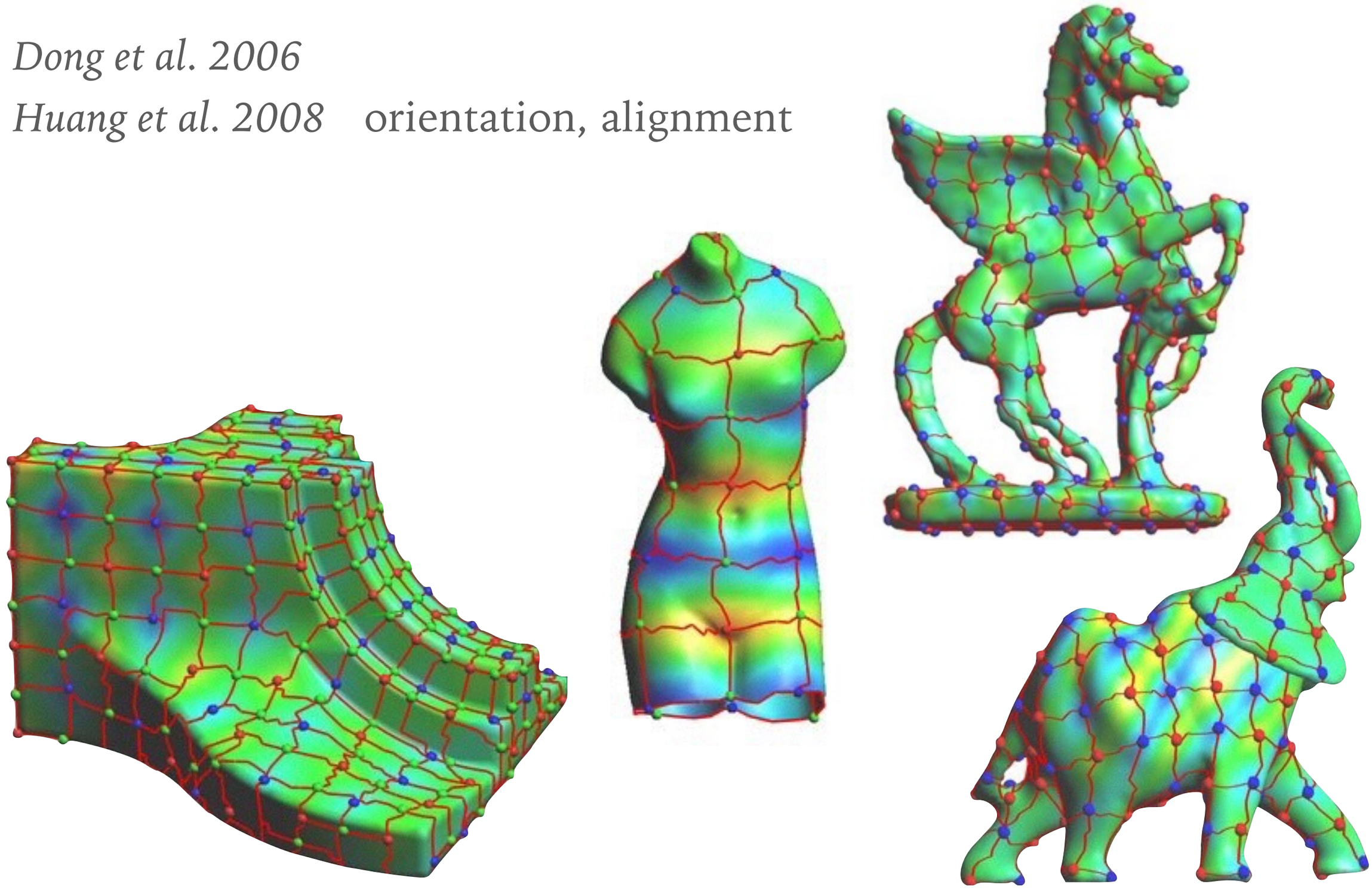

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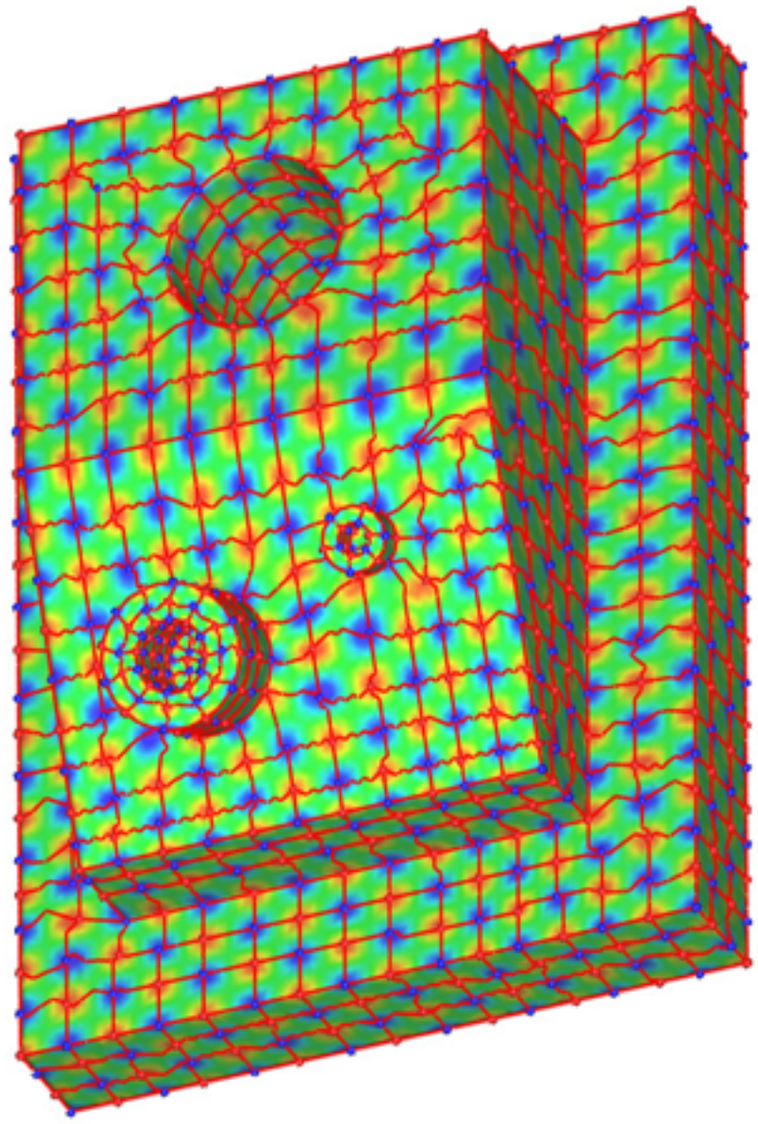

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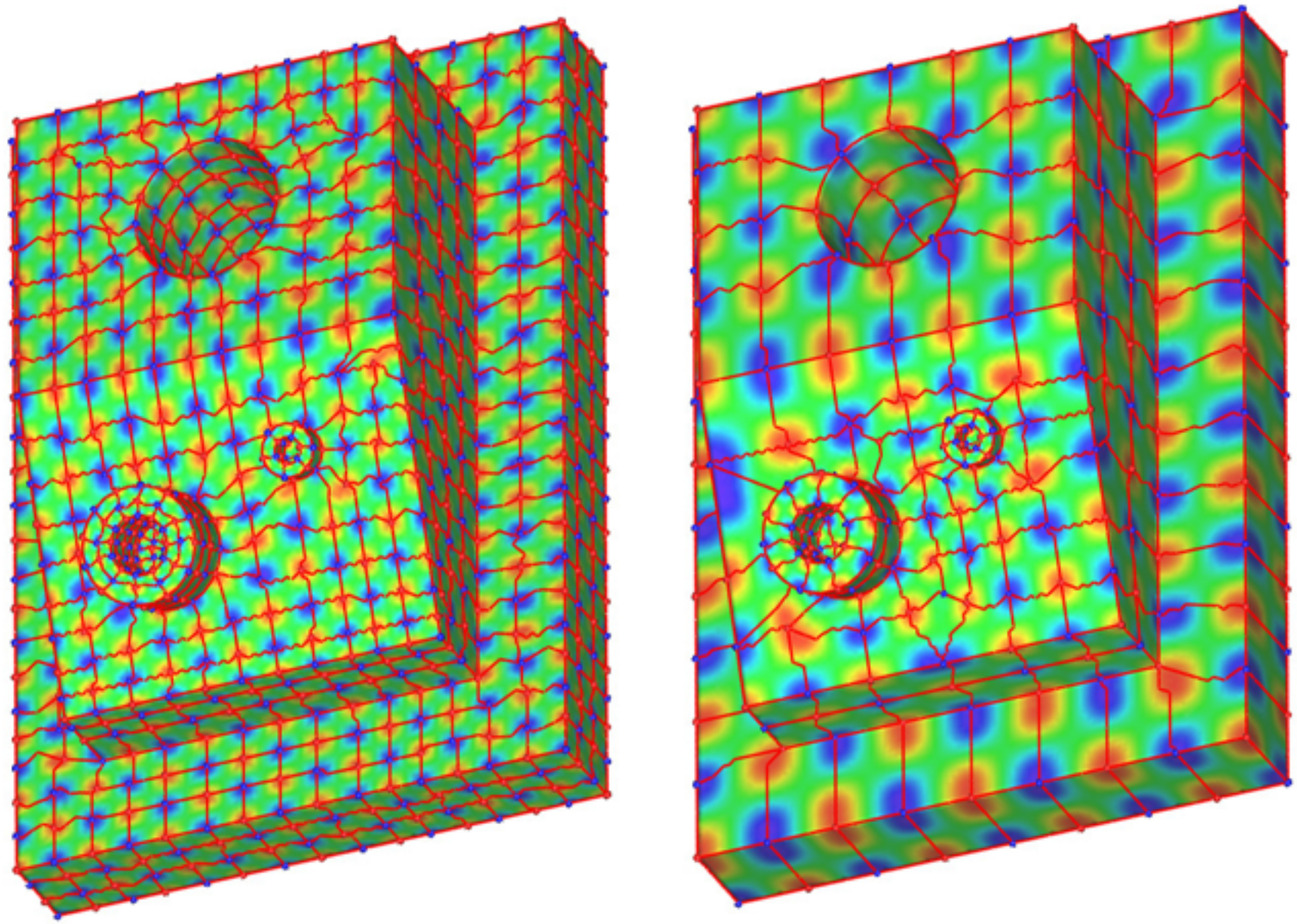

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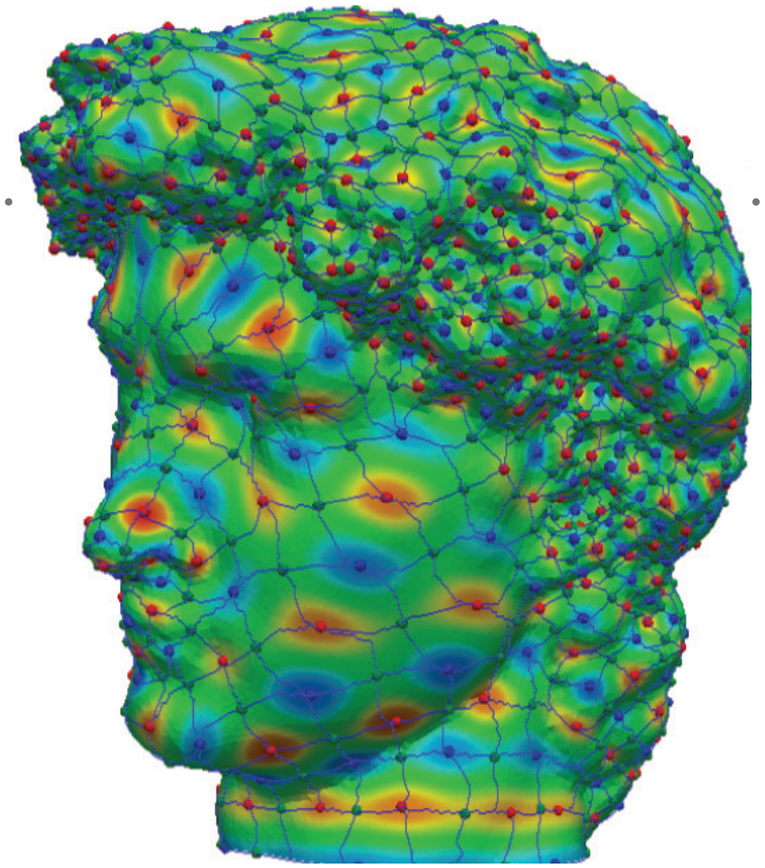

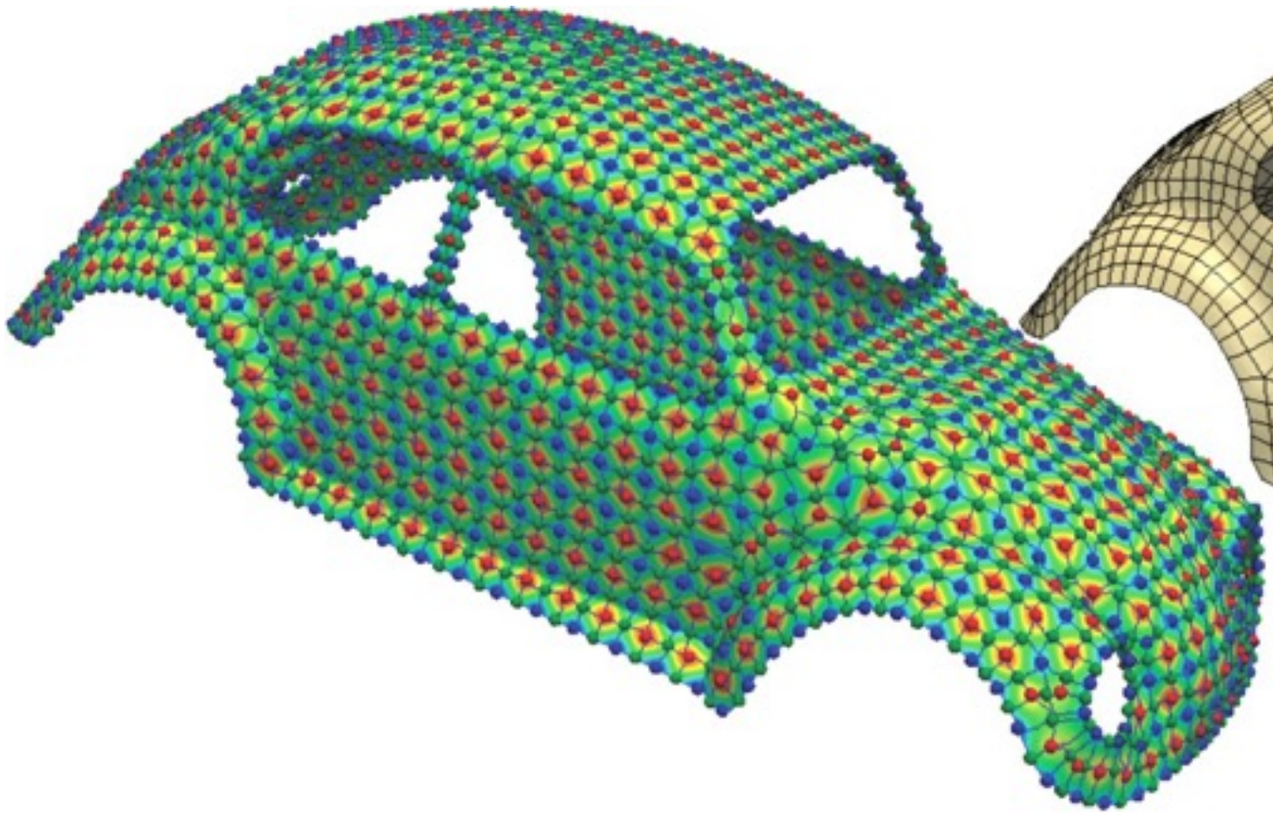

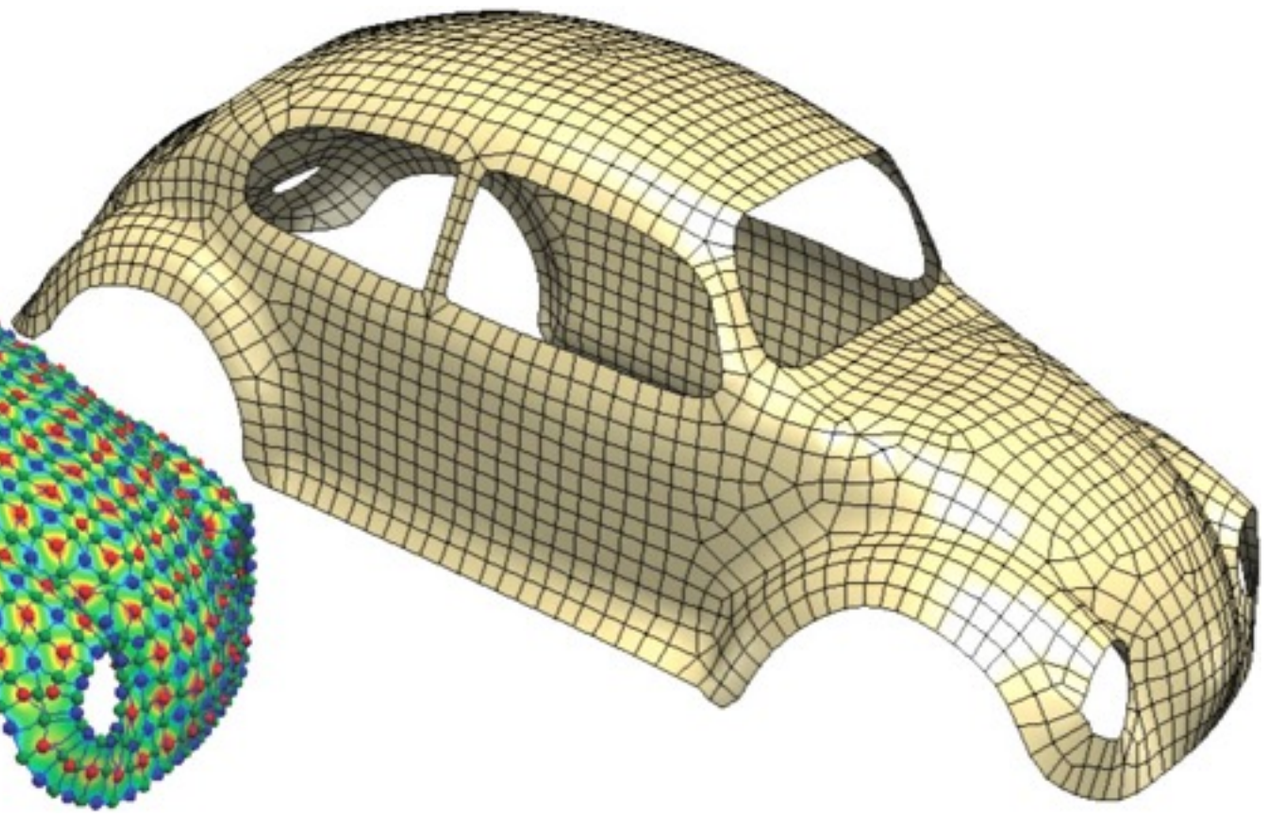

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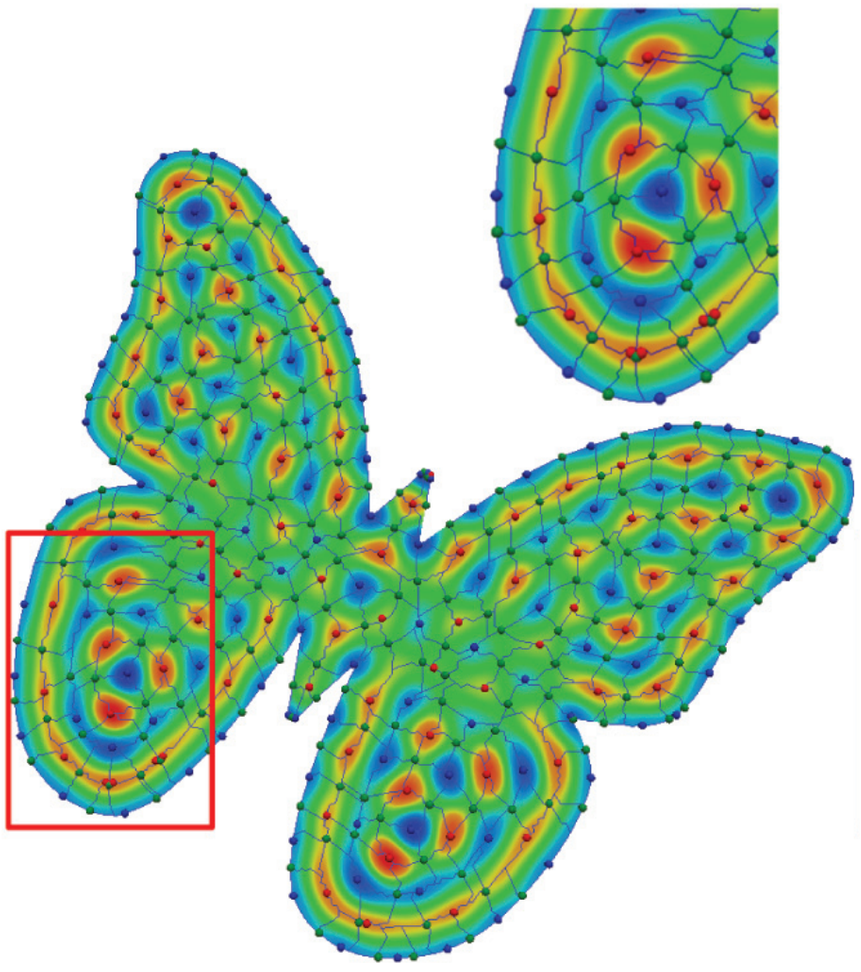
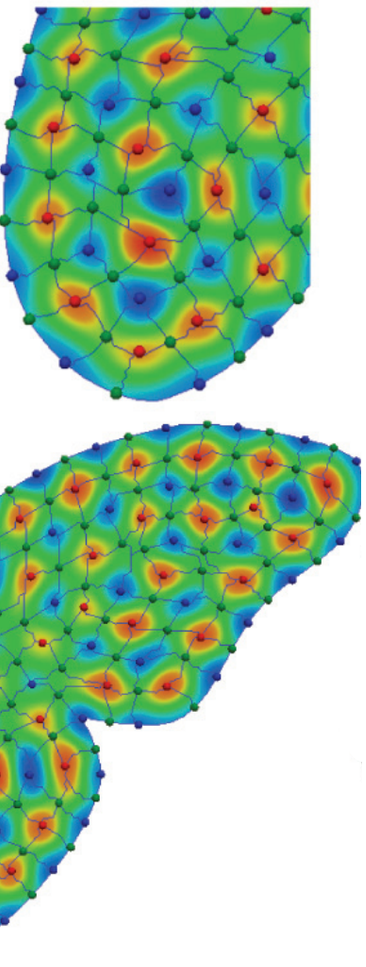
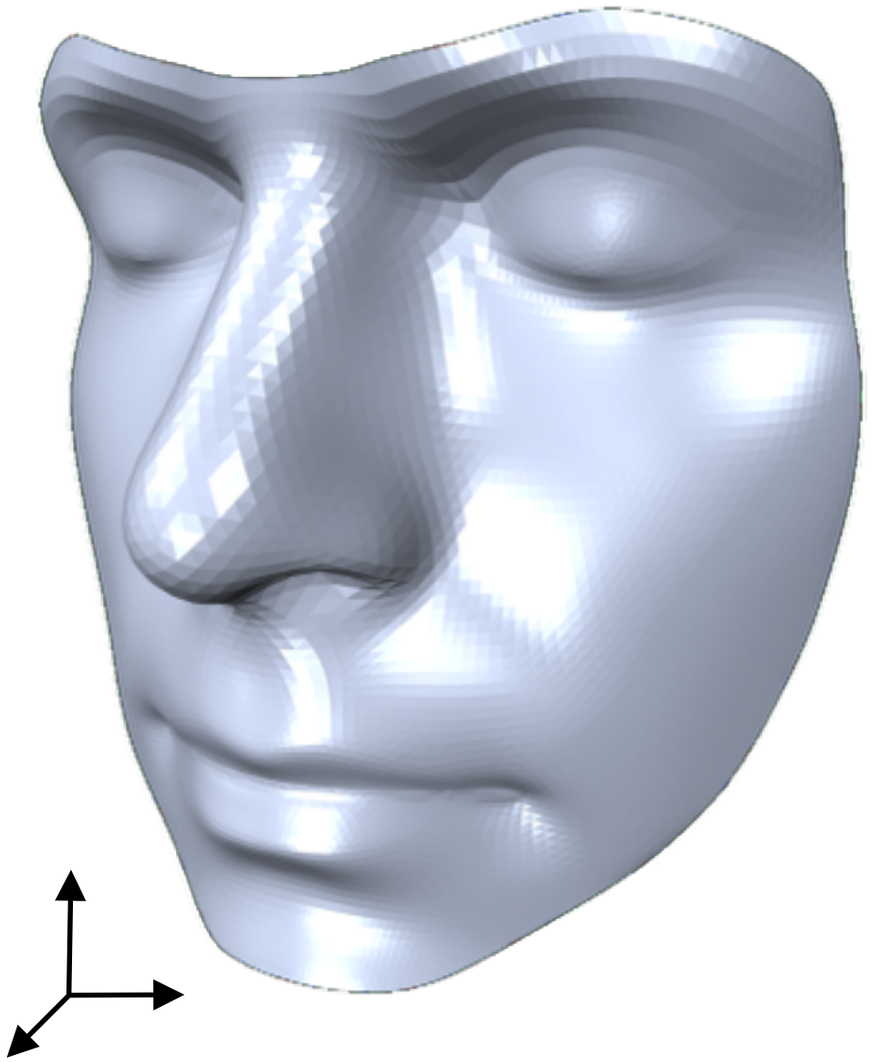




INTEGER GRID MAPS



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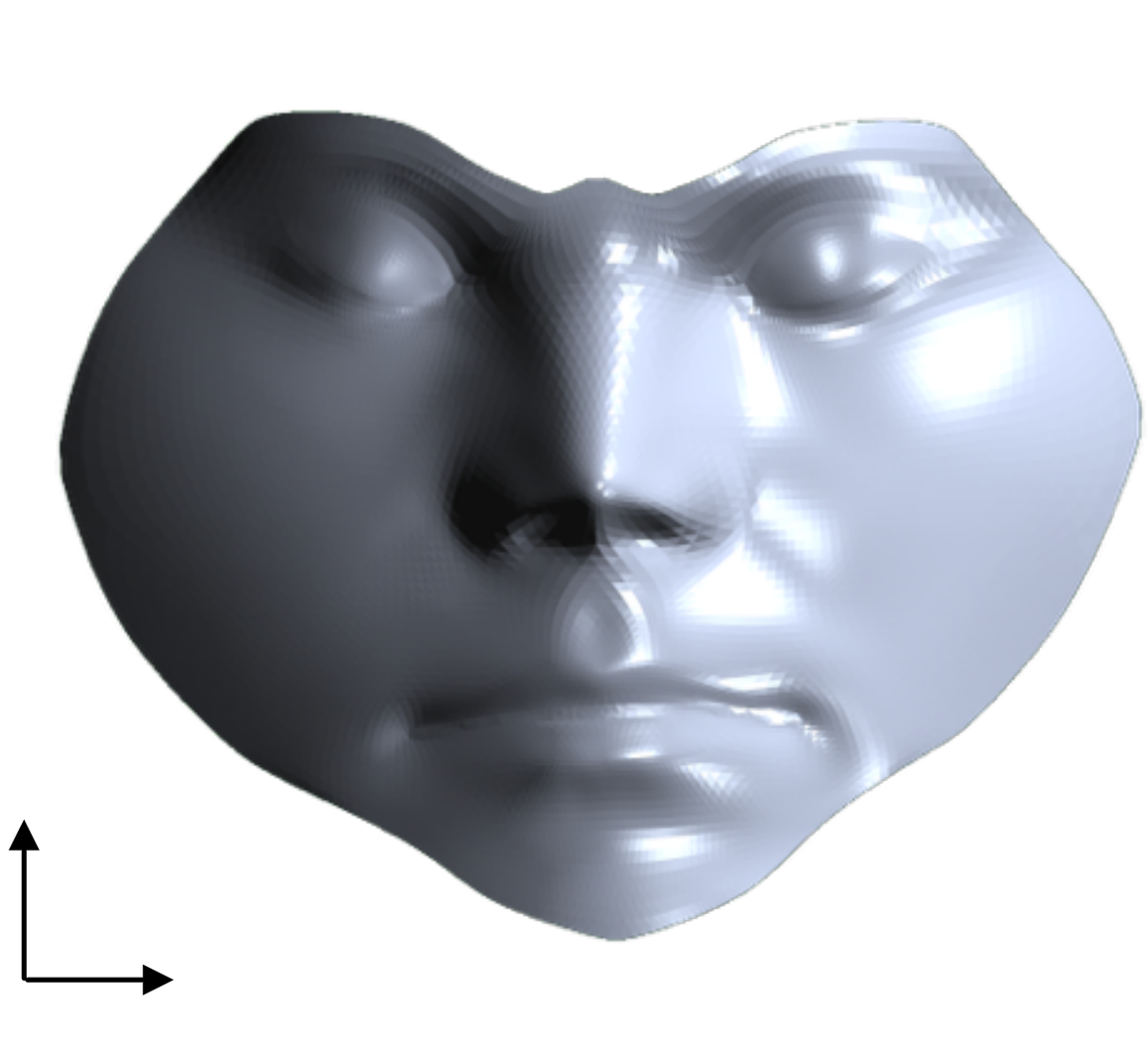

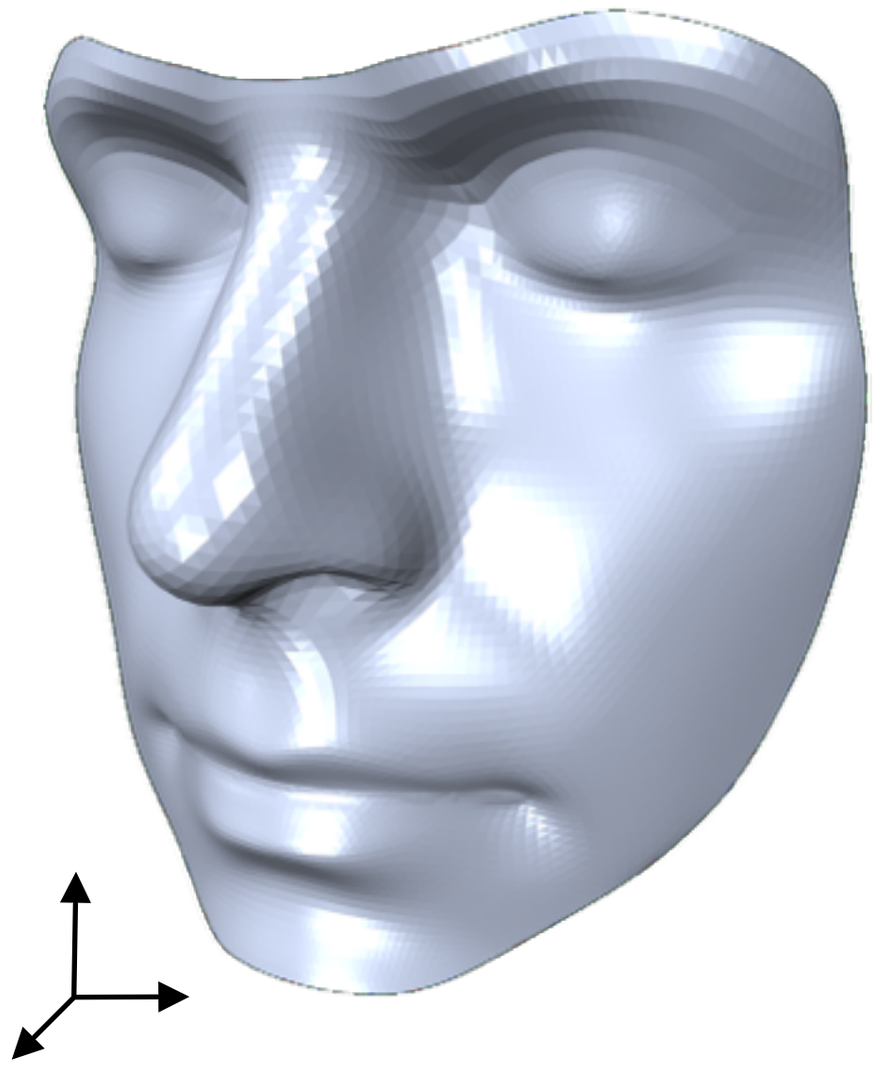

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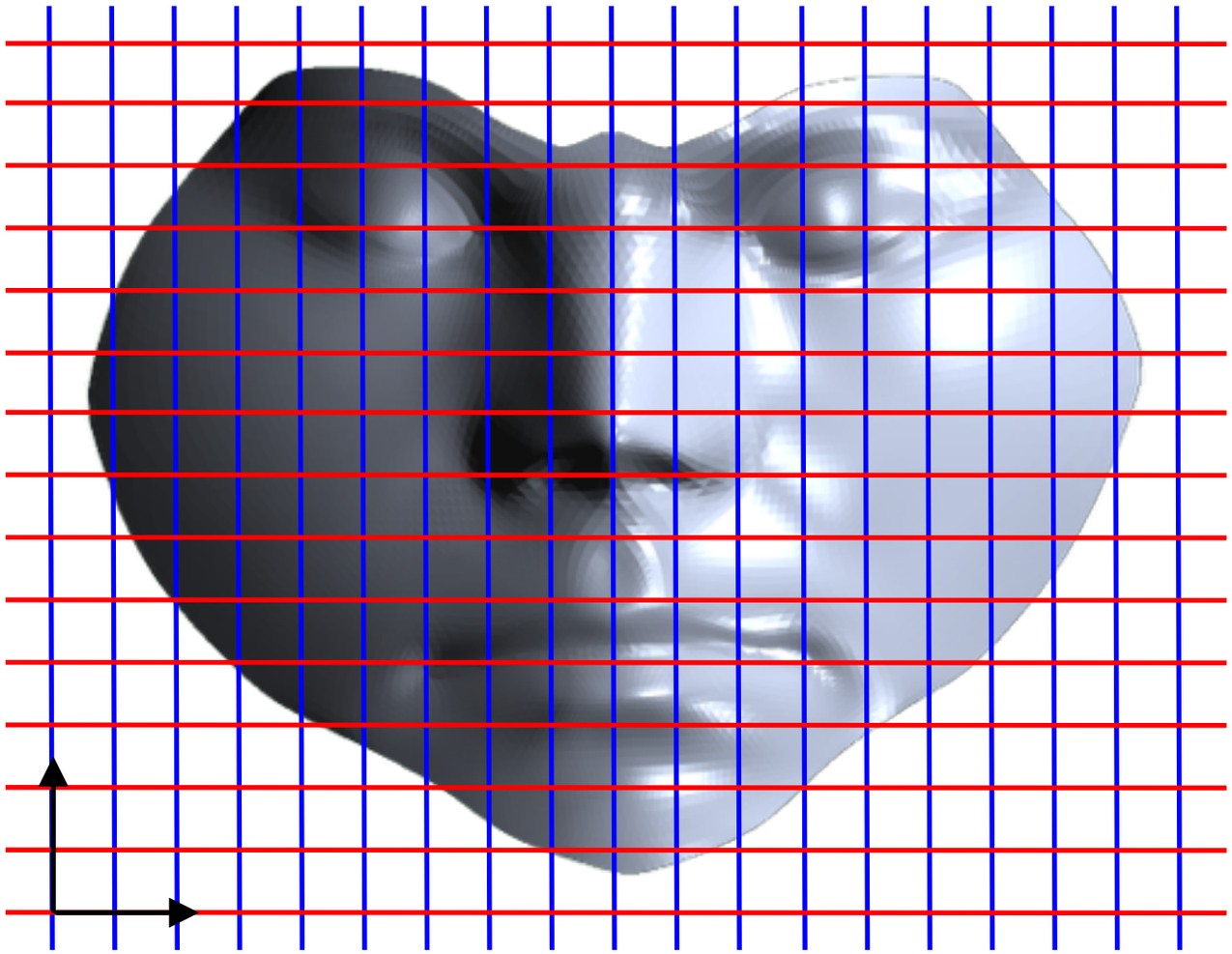

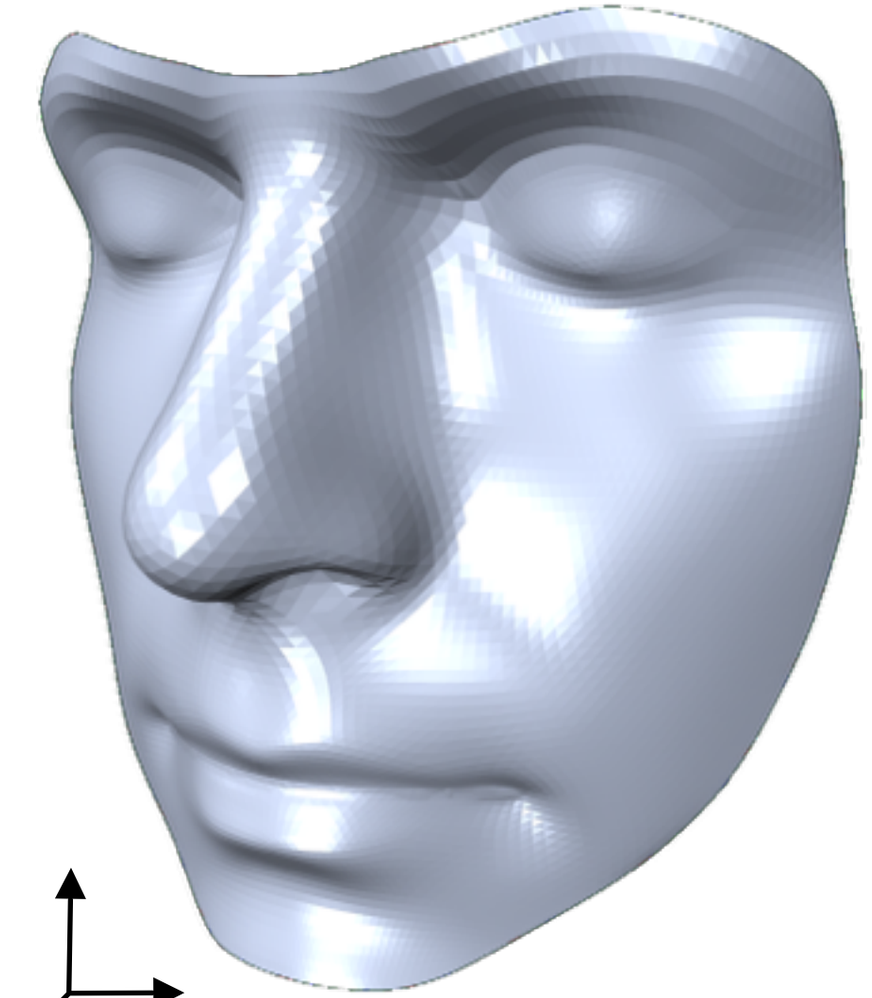

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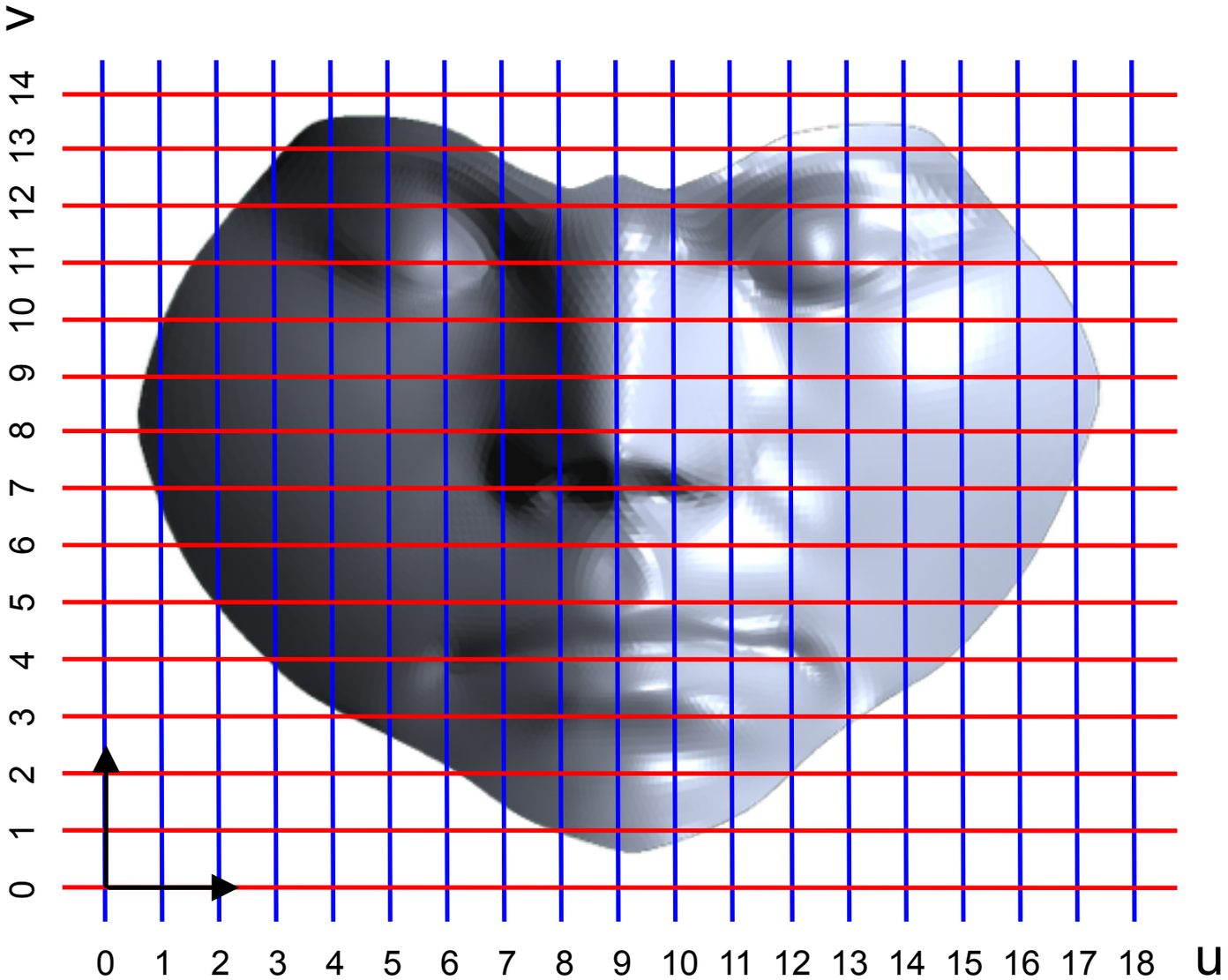

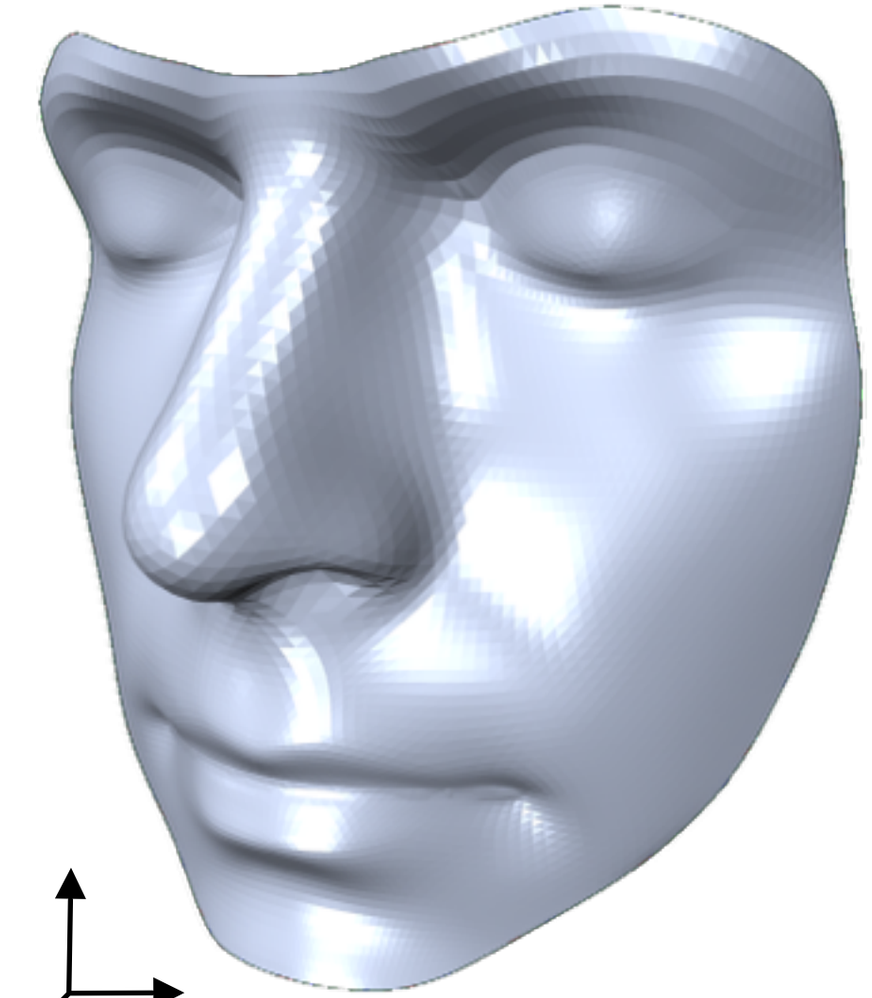

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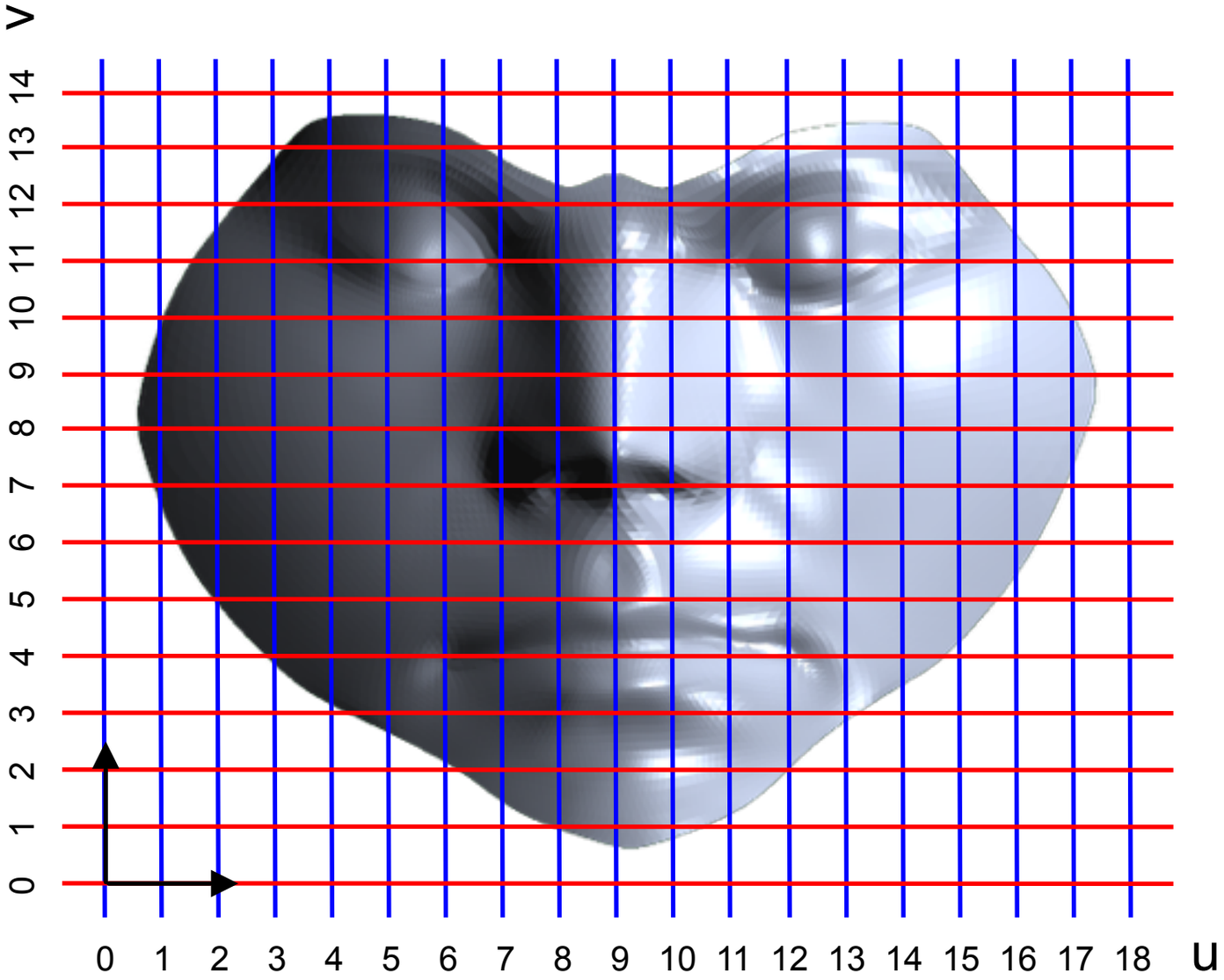

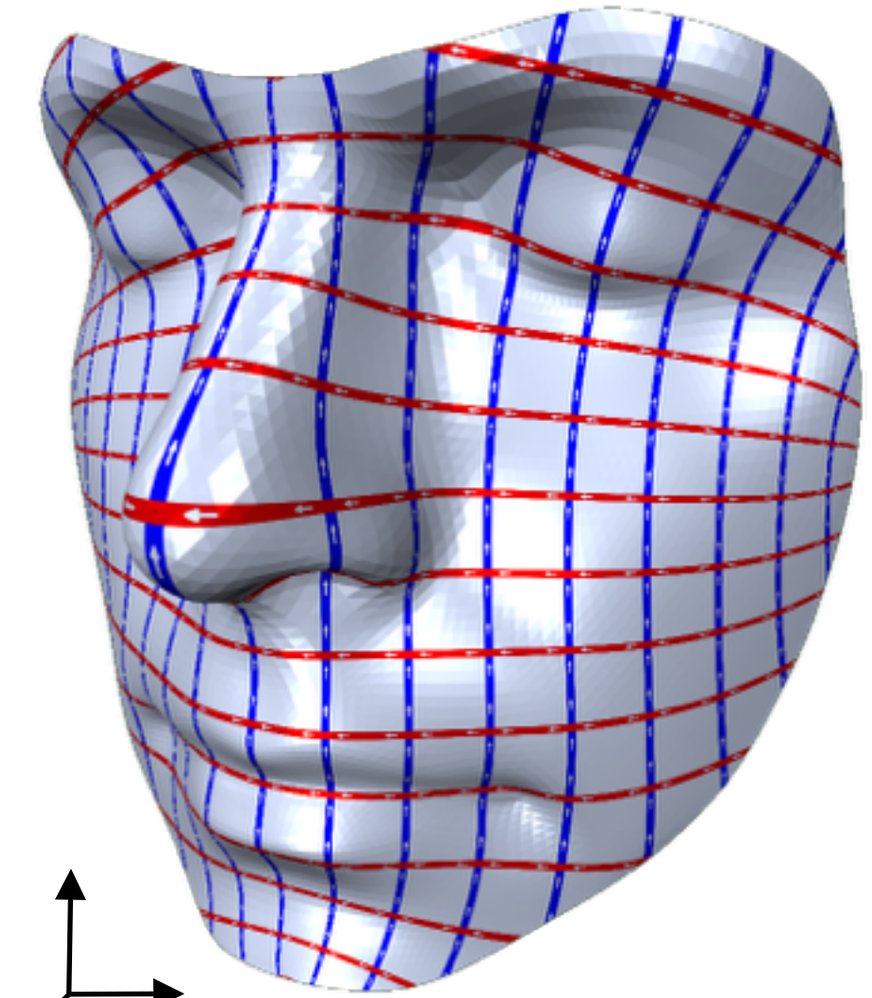

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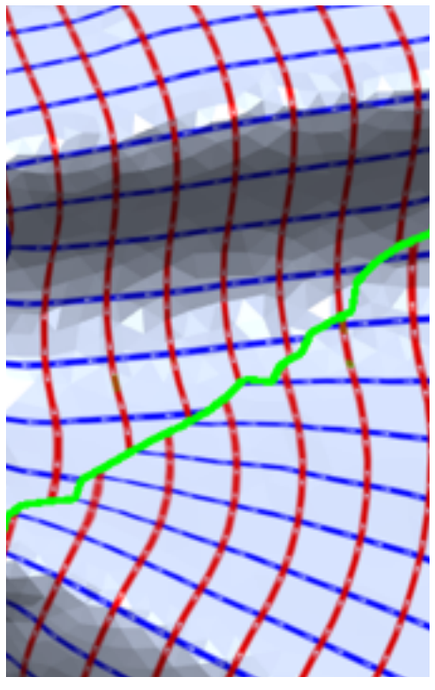

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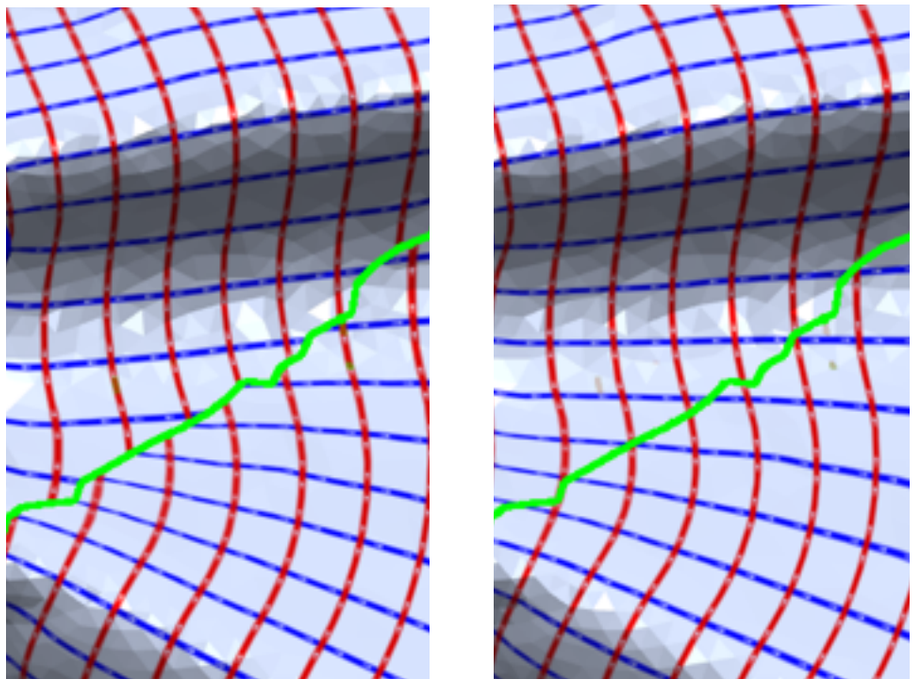

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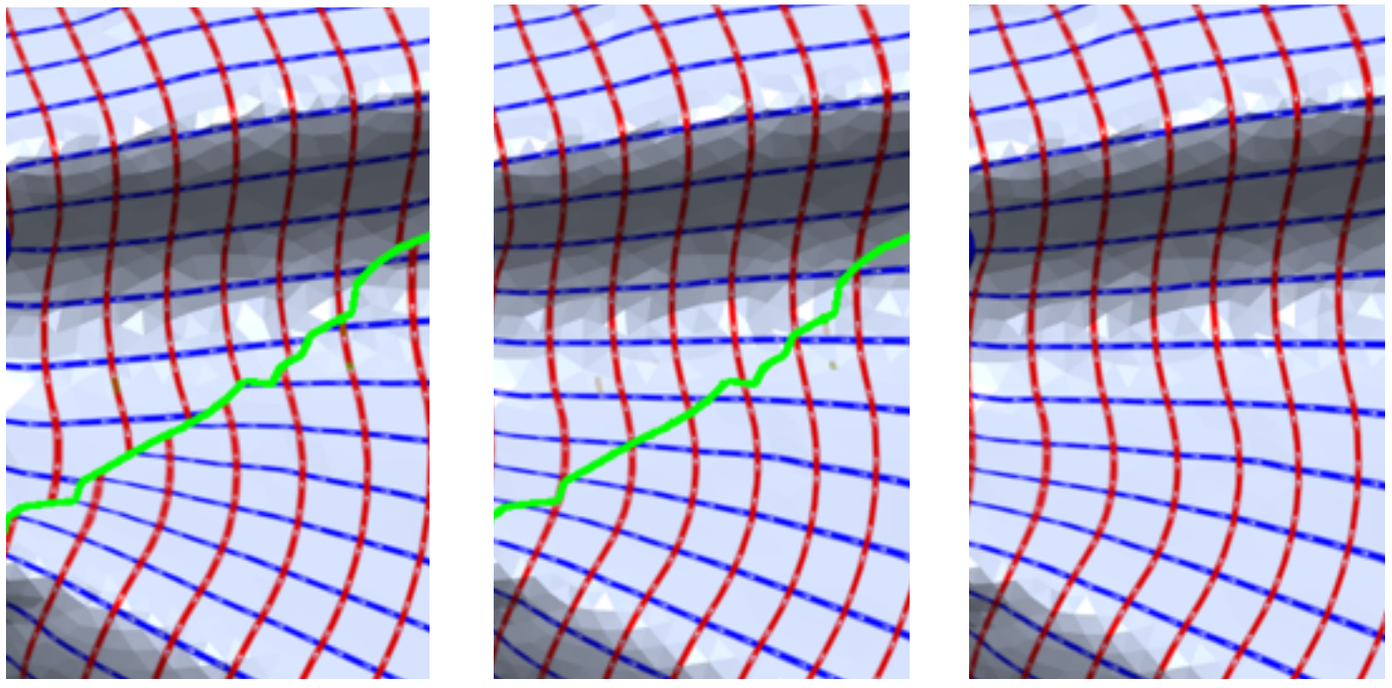

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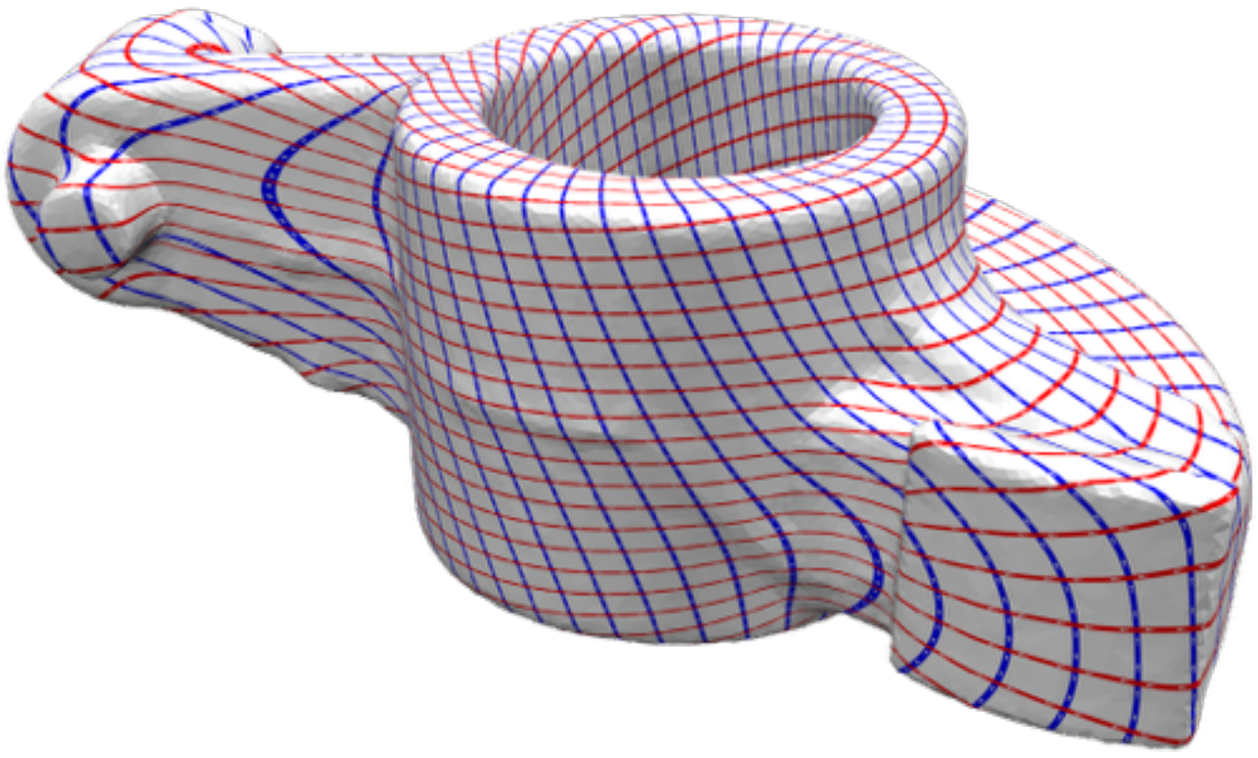

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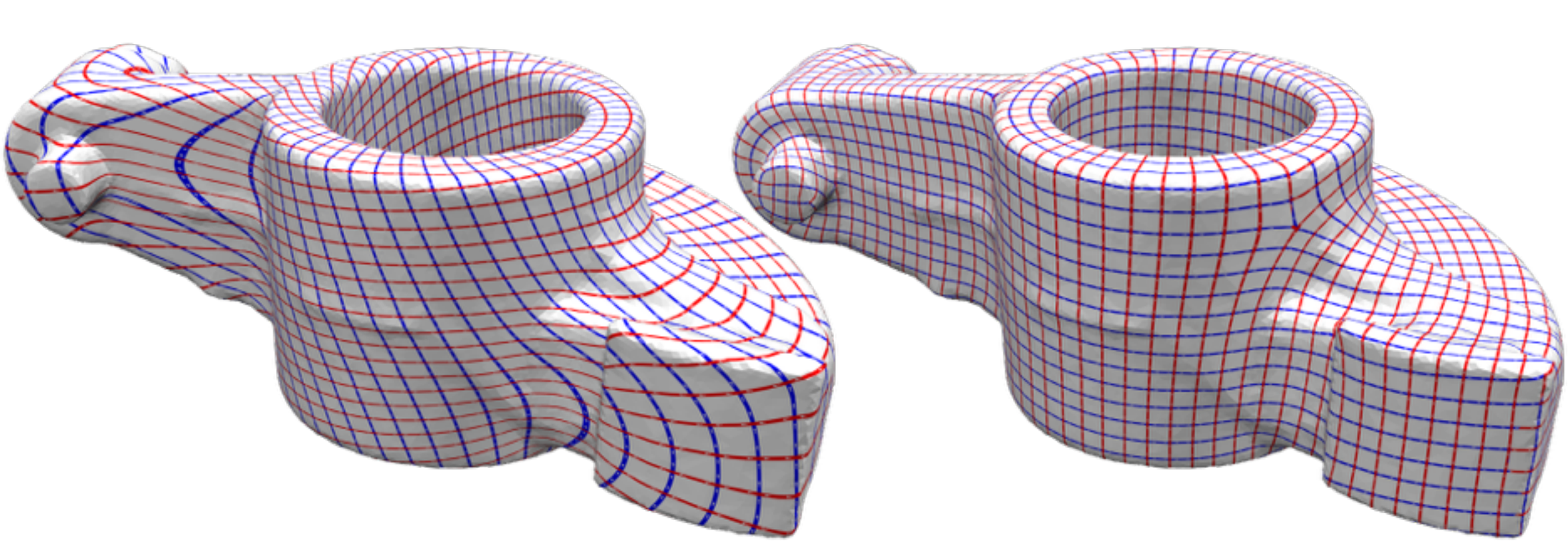

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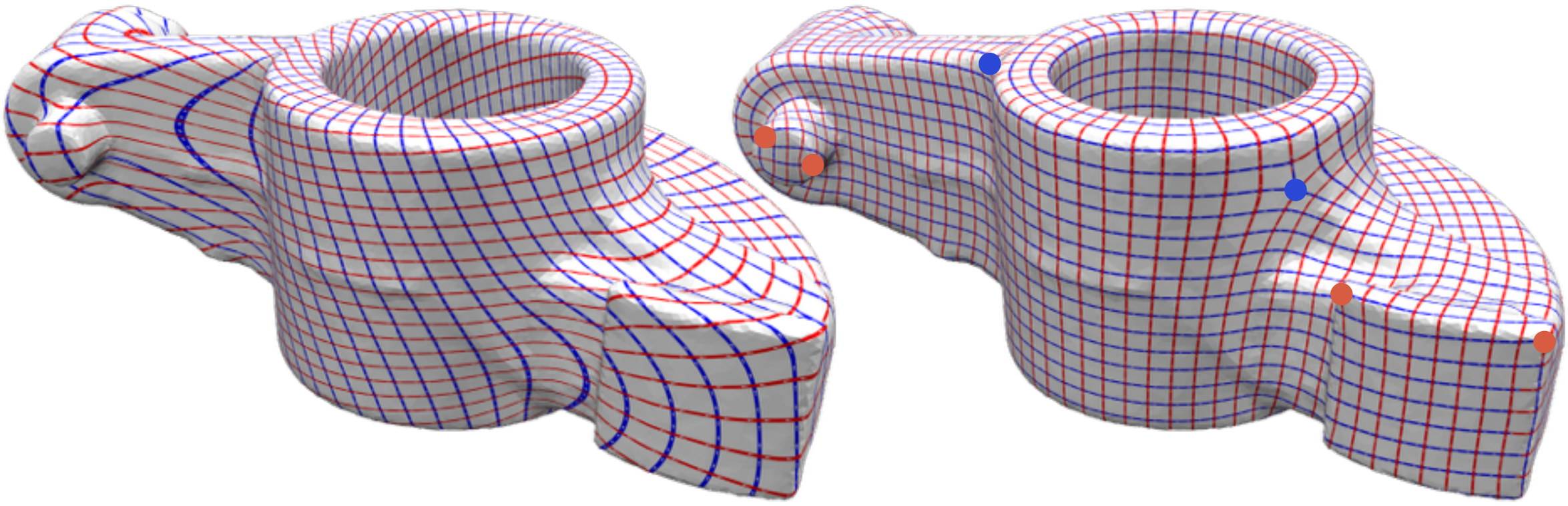

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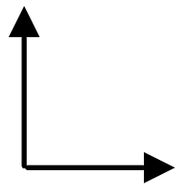

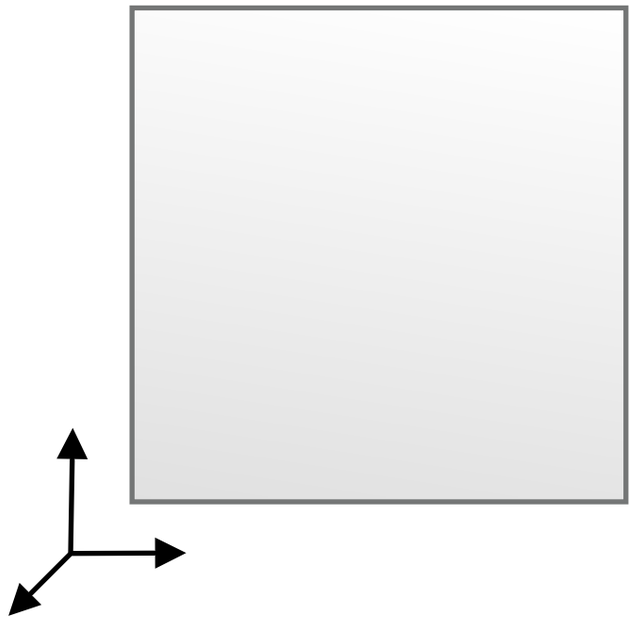

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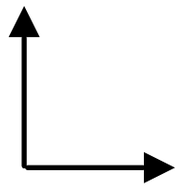

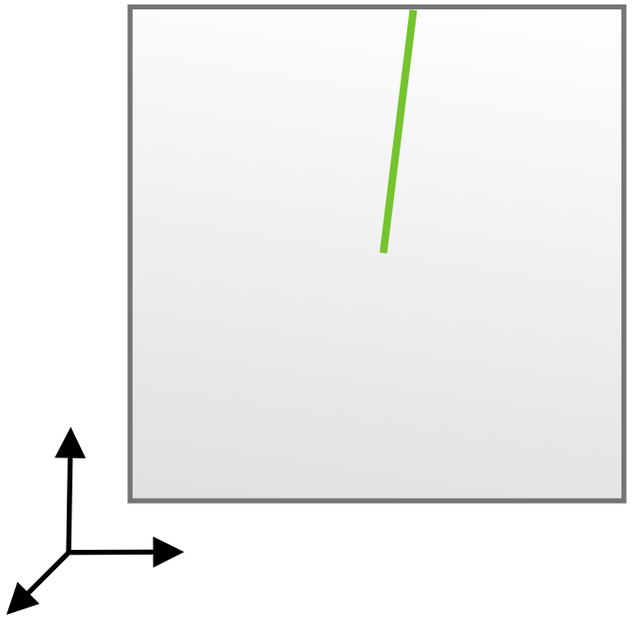

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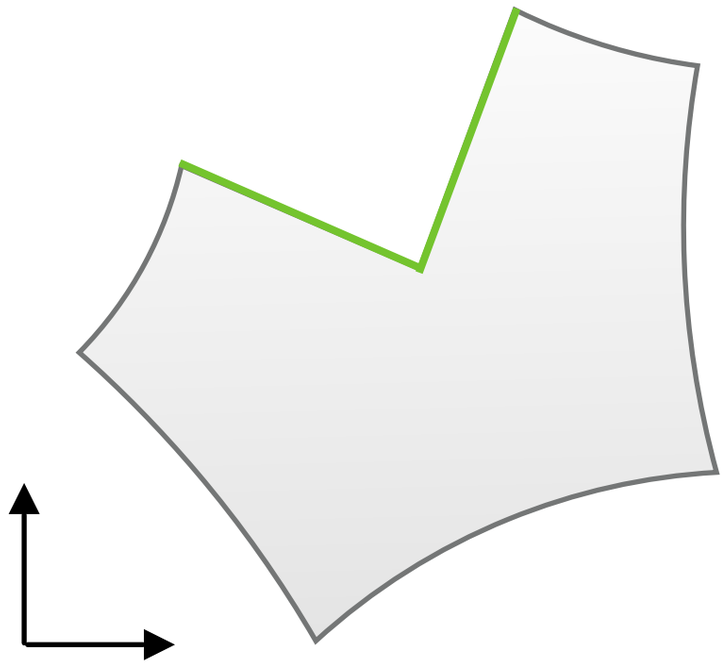
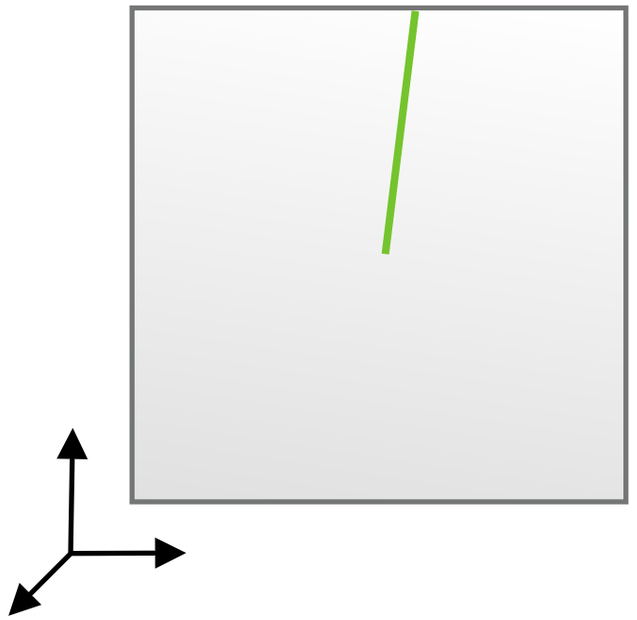
  - the grid in parameter space is completely regular





INTEGER GRID MAPS

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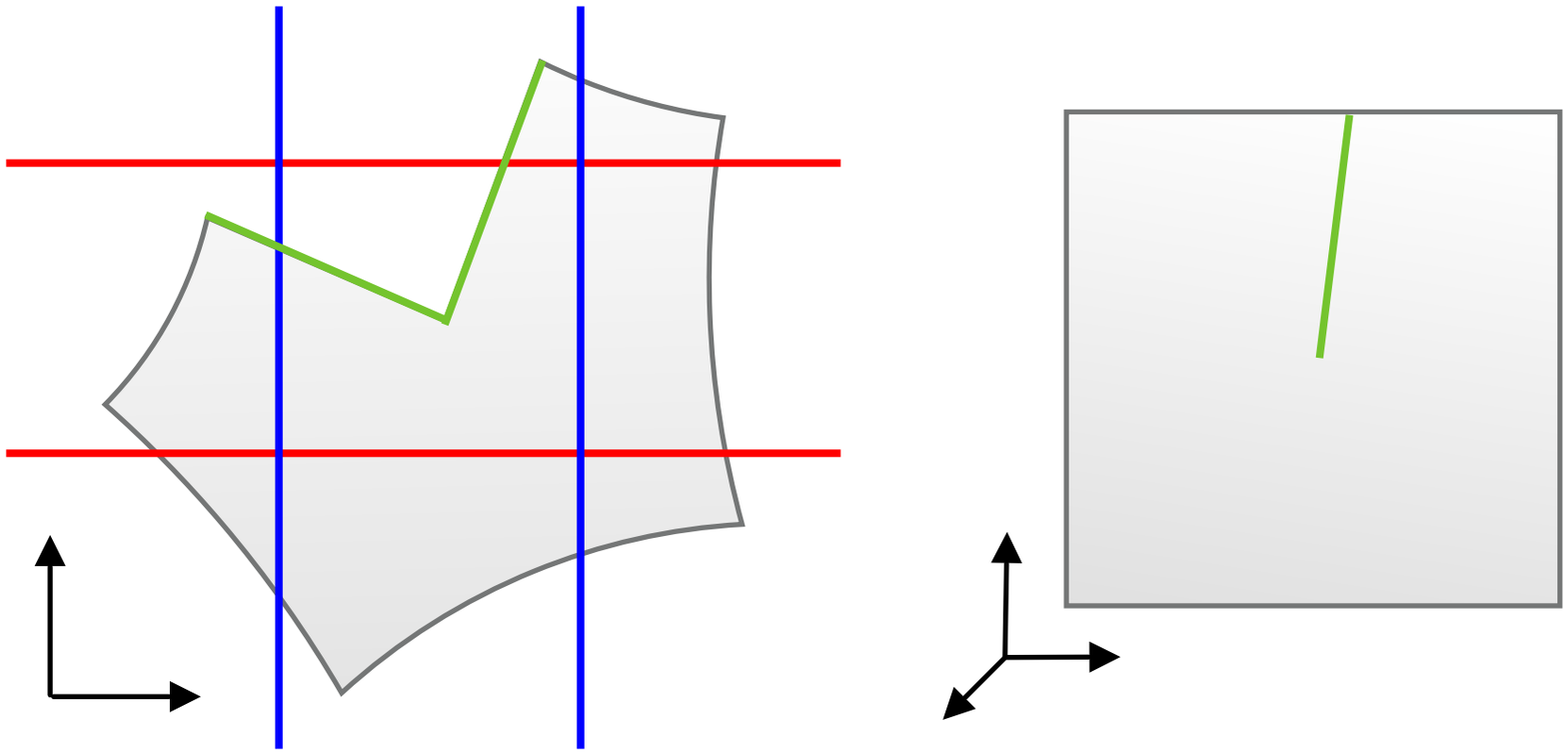

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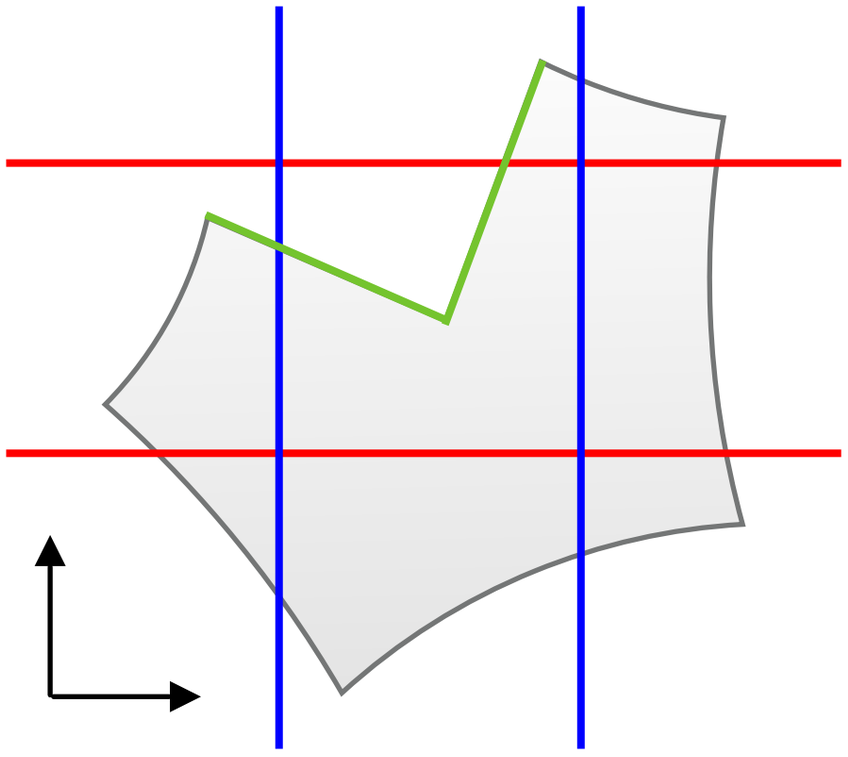

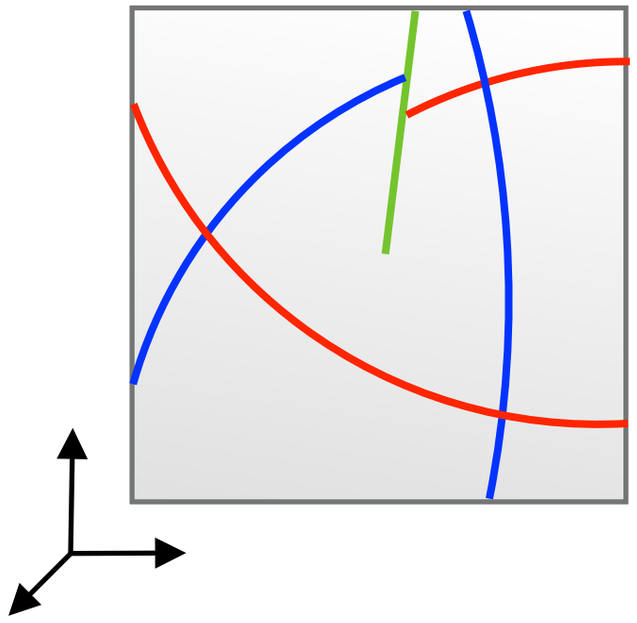

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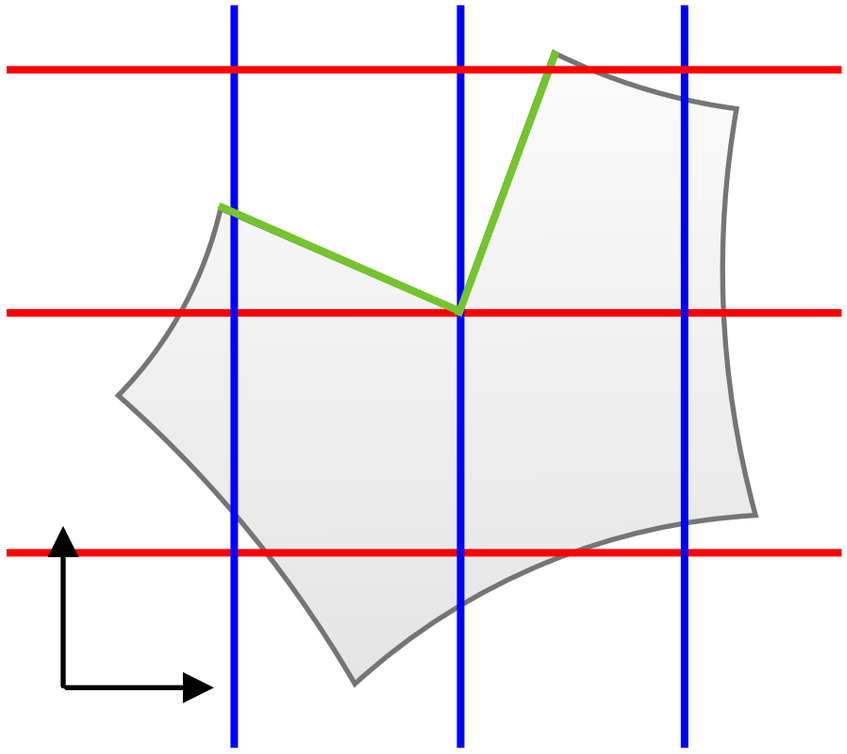

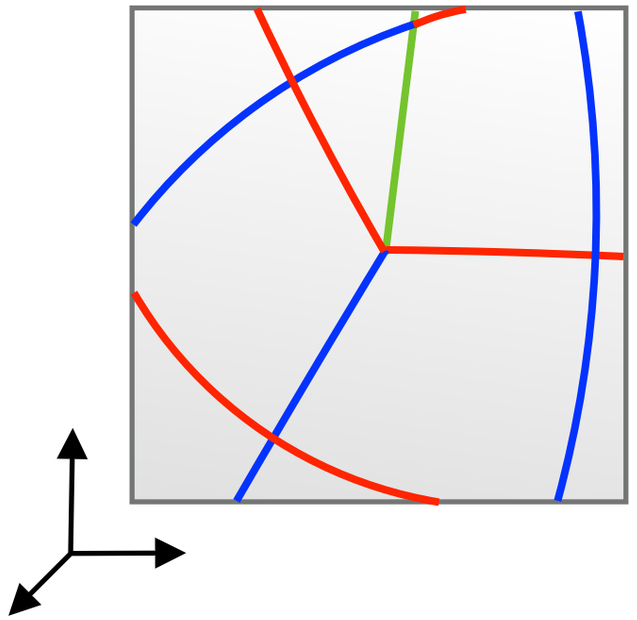

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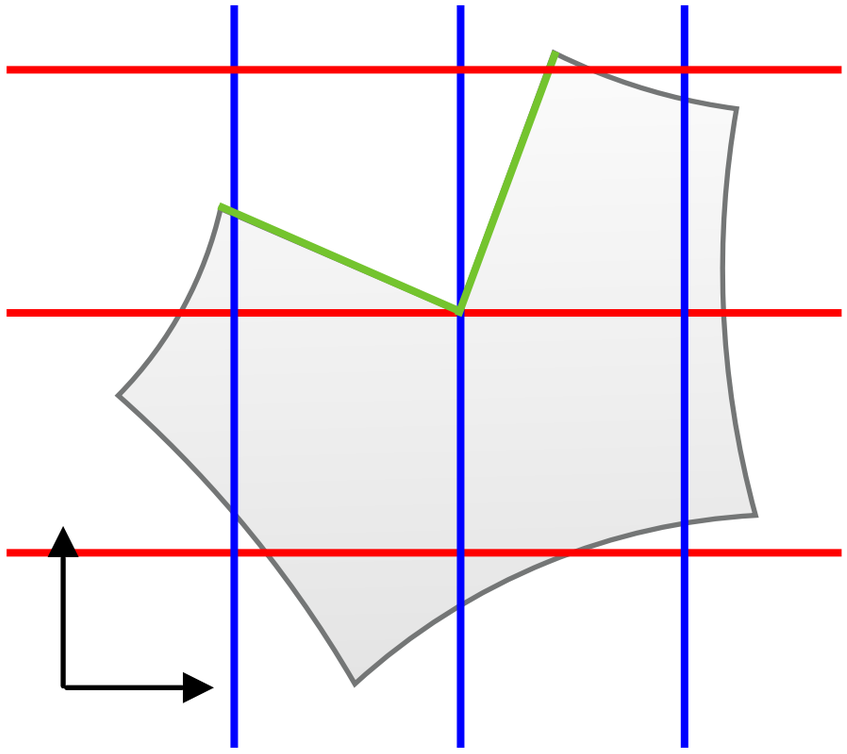
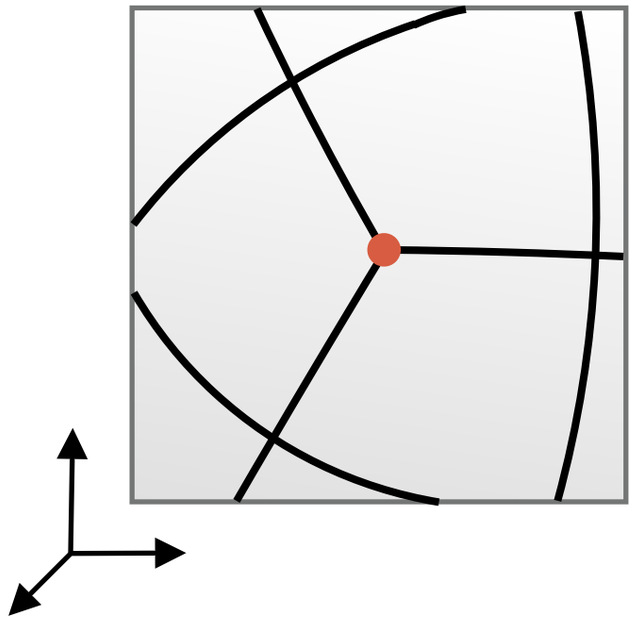




Kälberer et al. 2007
Bommes et al. 2009

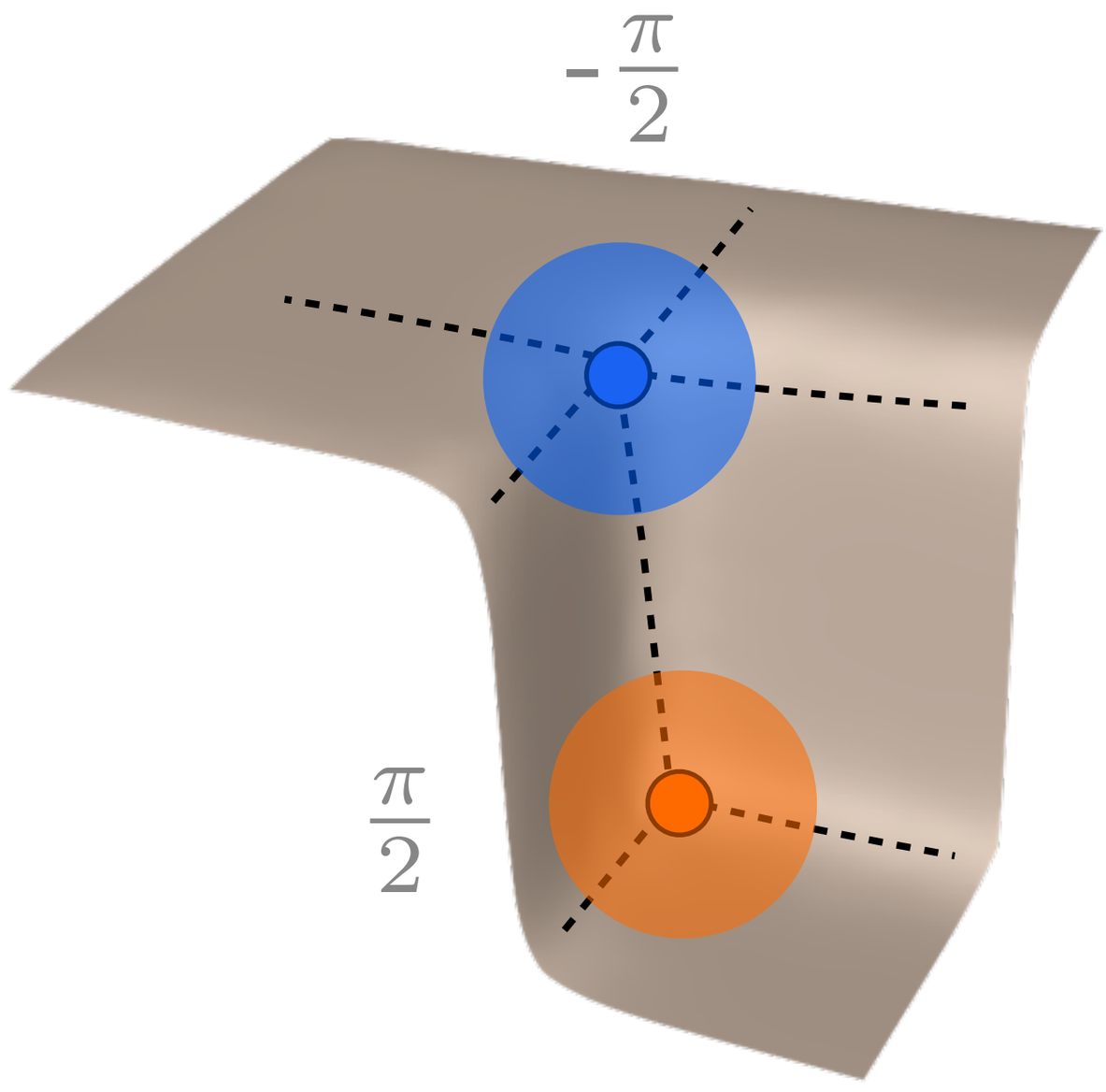
INTEGER GRID MAPS

- Problems:
  - supports only topological disks (and punctured disks)
    - more complex surfaces: cut open, constrained transitions
  - the grid in parameter space is completely regular
    - irregular vertices: cut, constrained transitions ($k \cdot 90°$)

INTEGER GRID MAPS

- Problems:
  - supports only topological disks (and punctured disks)
    - more complex surfaces: cut open, constrained transitions
  - the grid in parameter space is completely regular
    - irregular vertices: cut, constrained transitions ($k \cdot 90°$)
    - where to put irregular vertex with which valence?

IRREGULAR VERTICES



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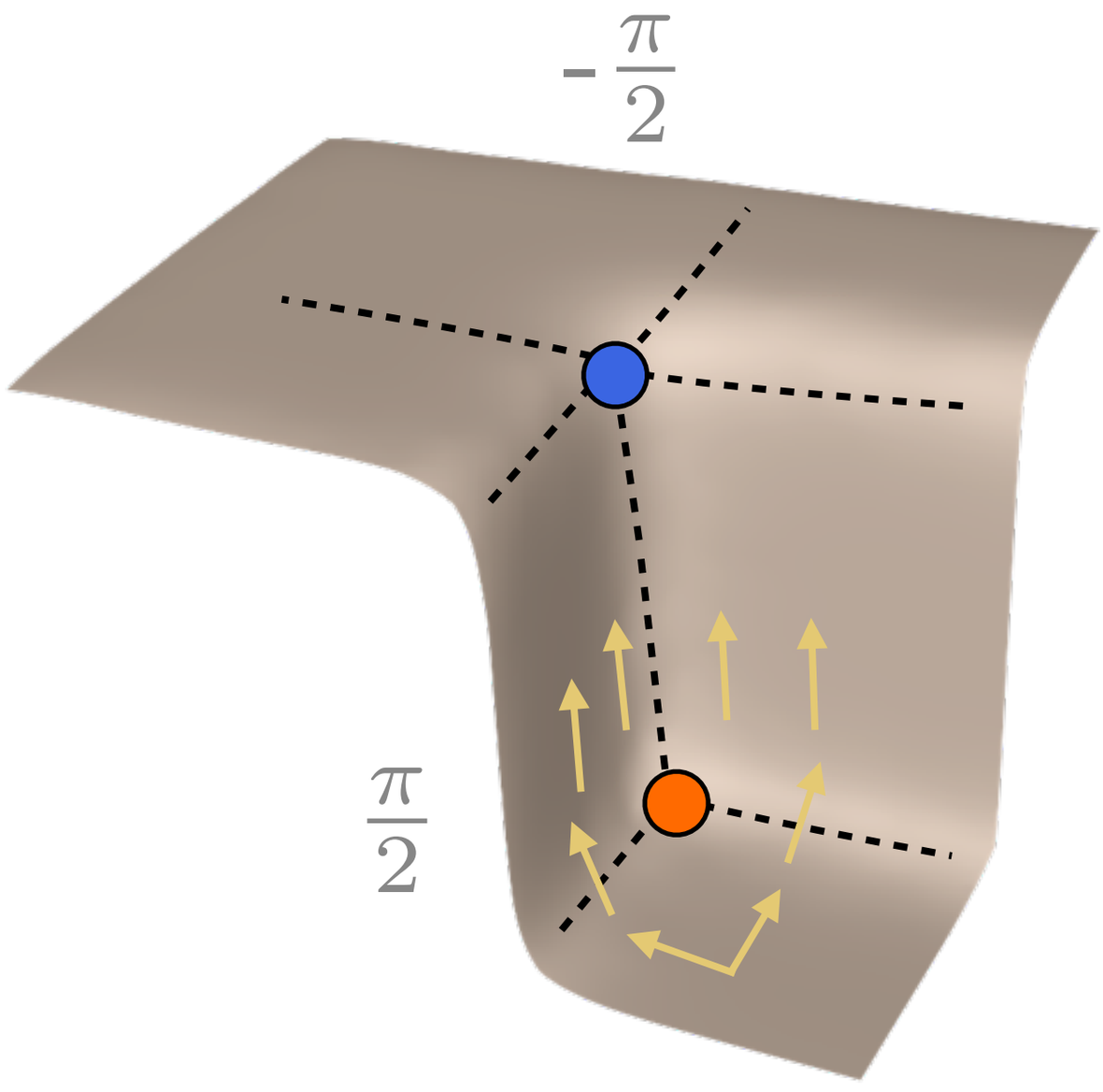

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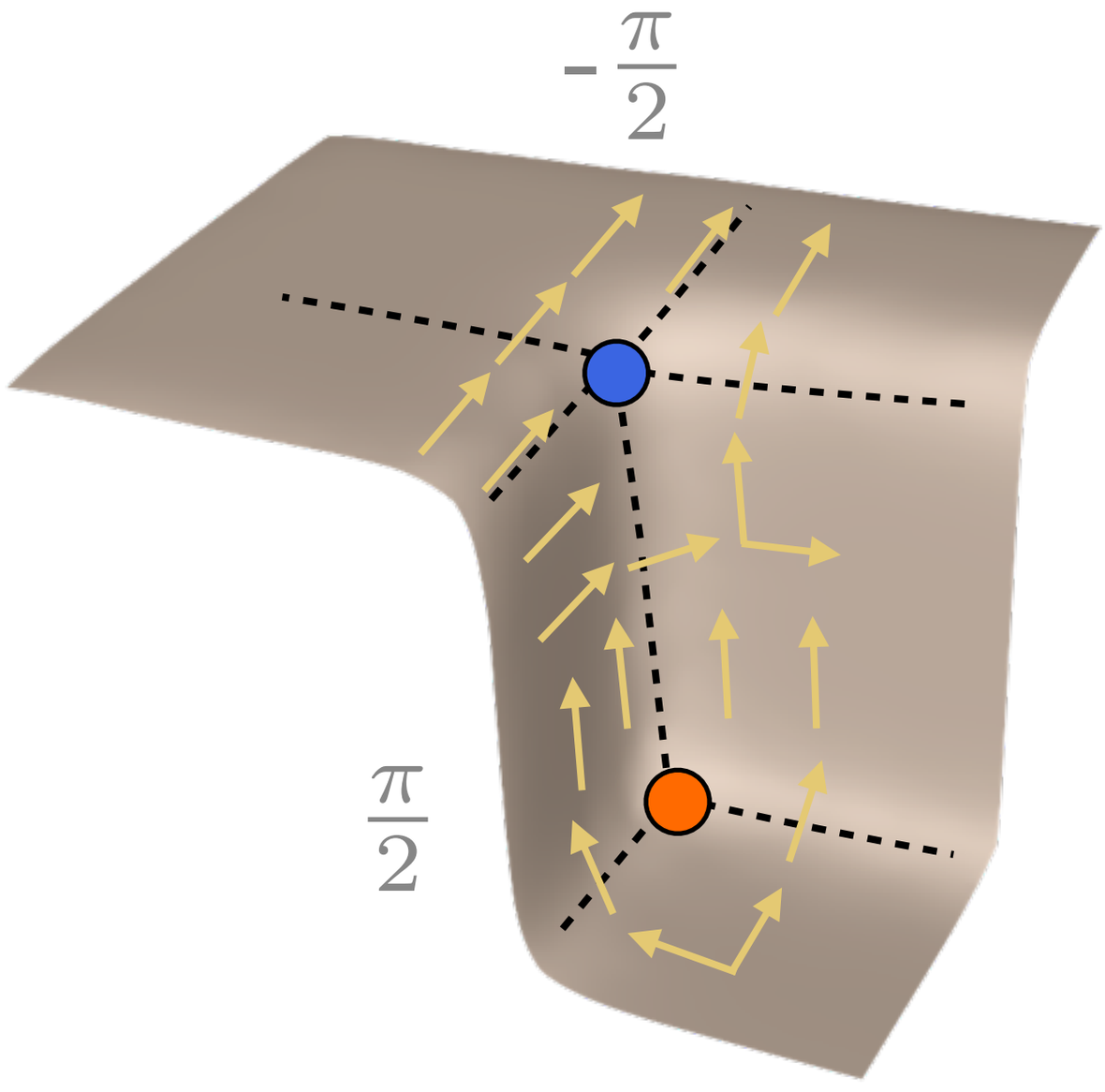

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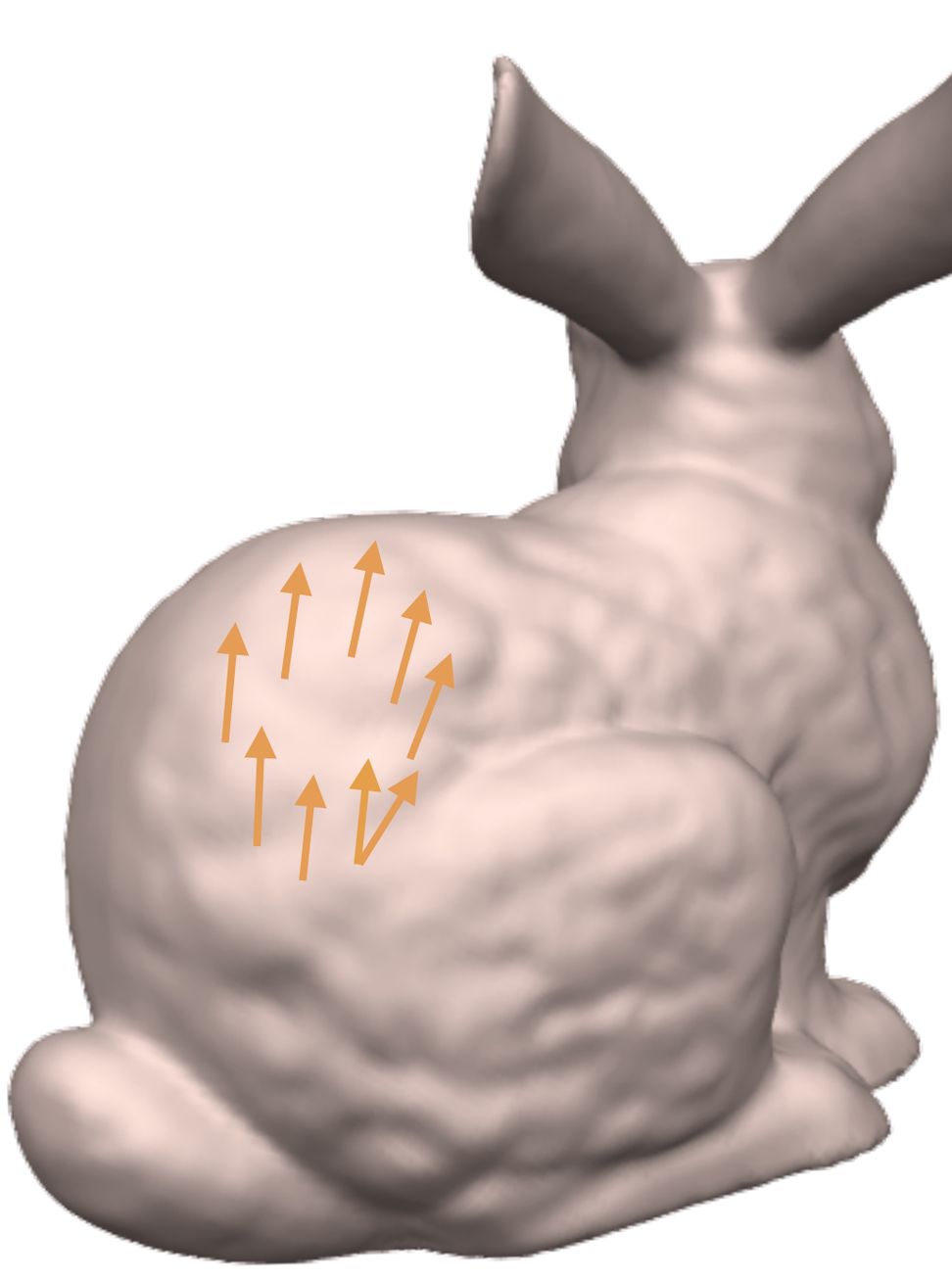

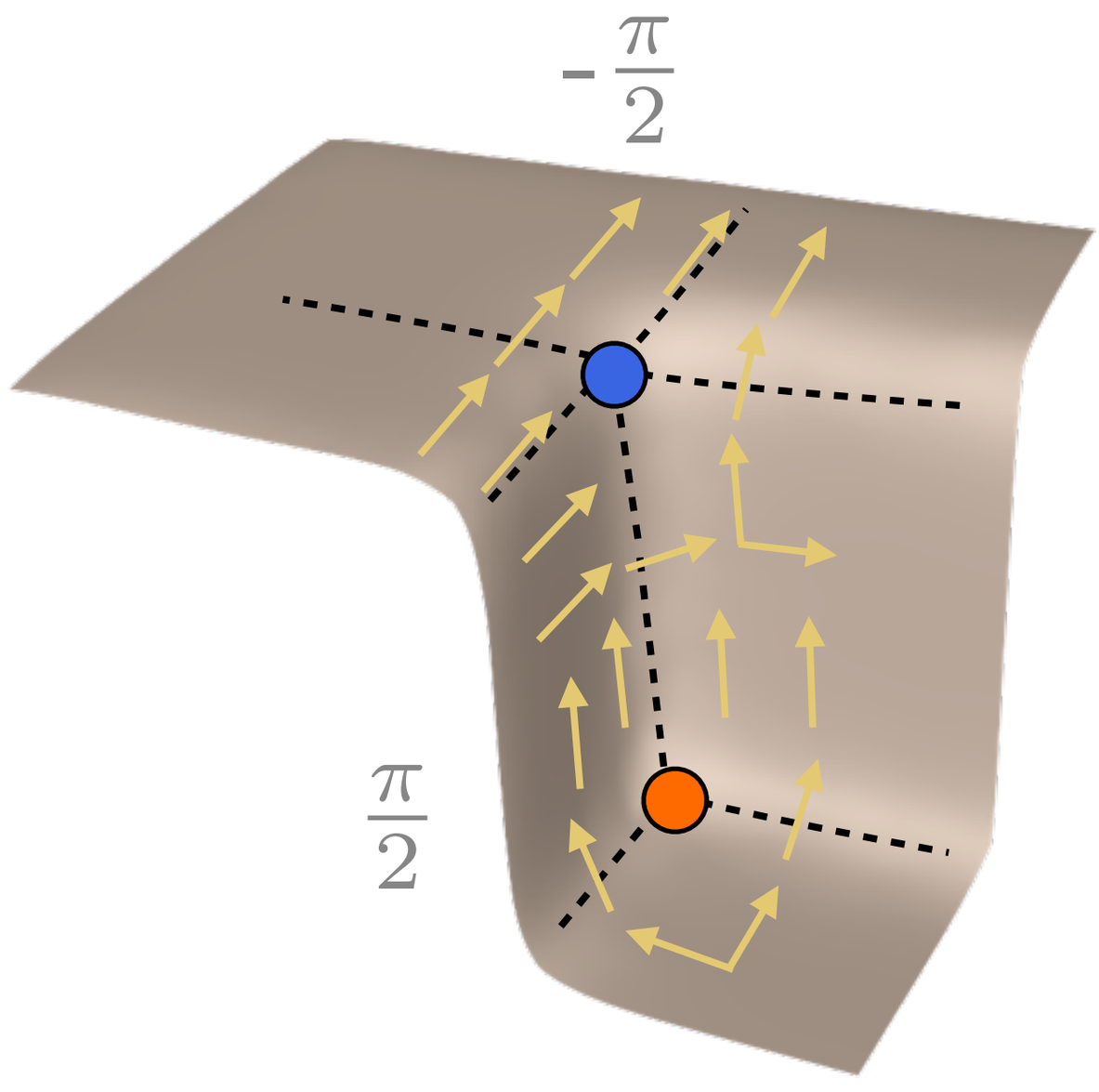

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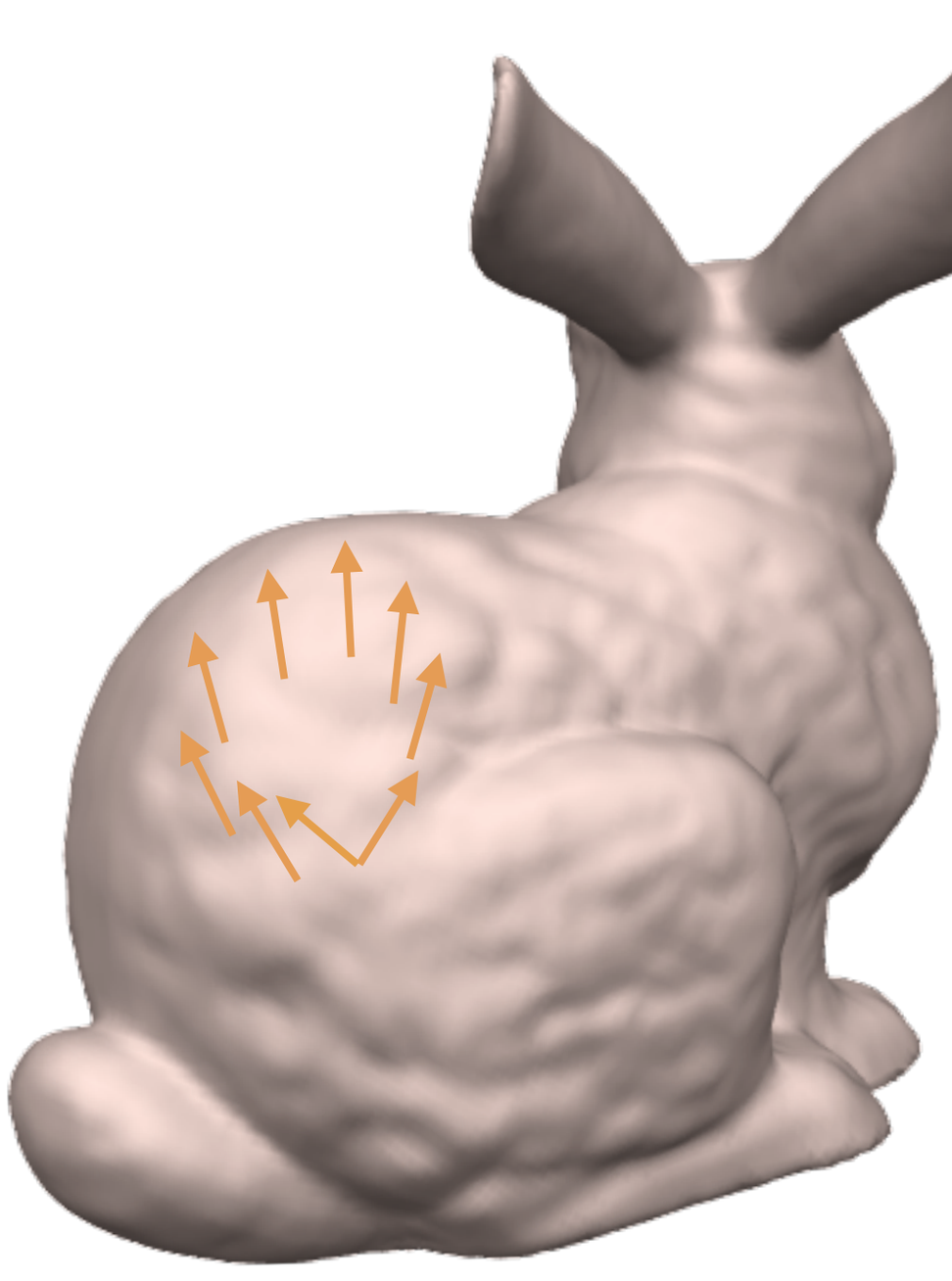
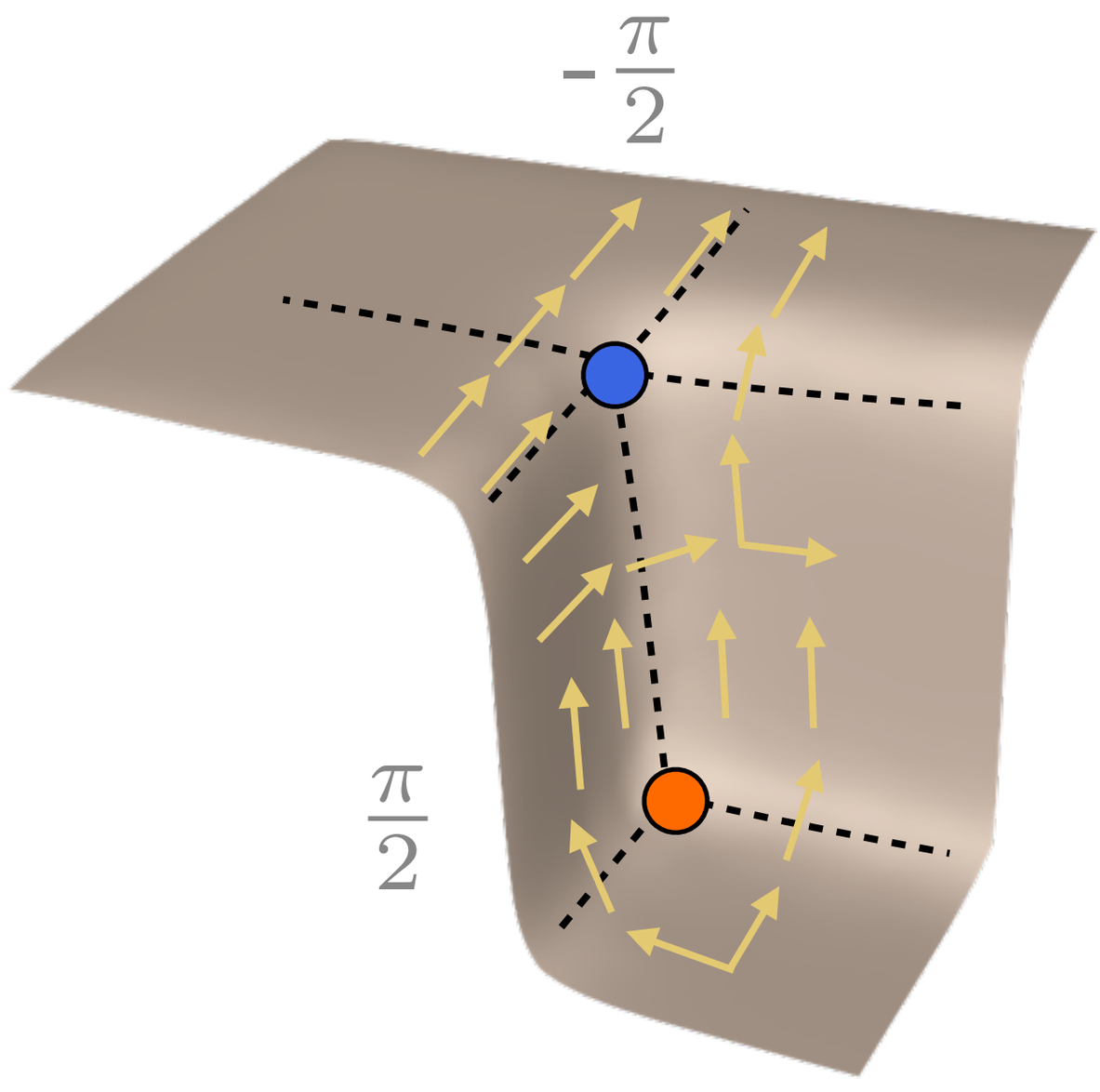
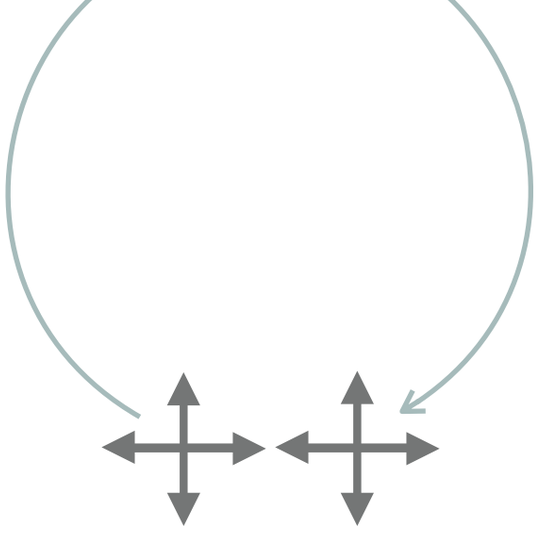
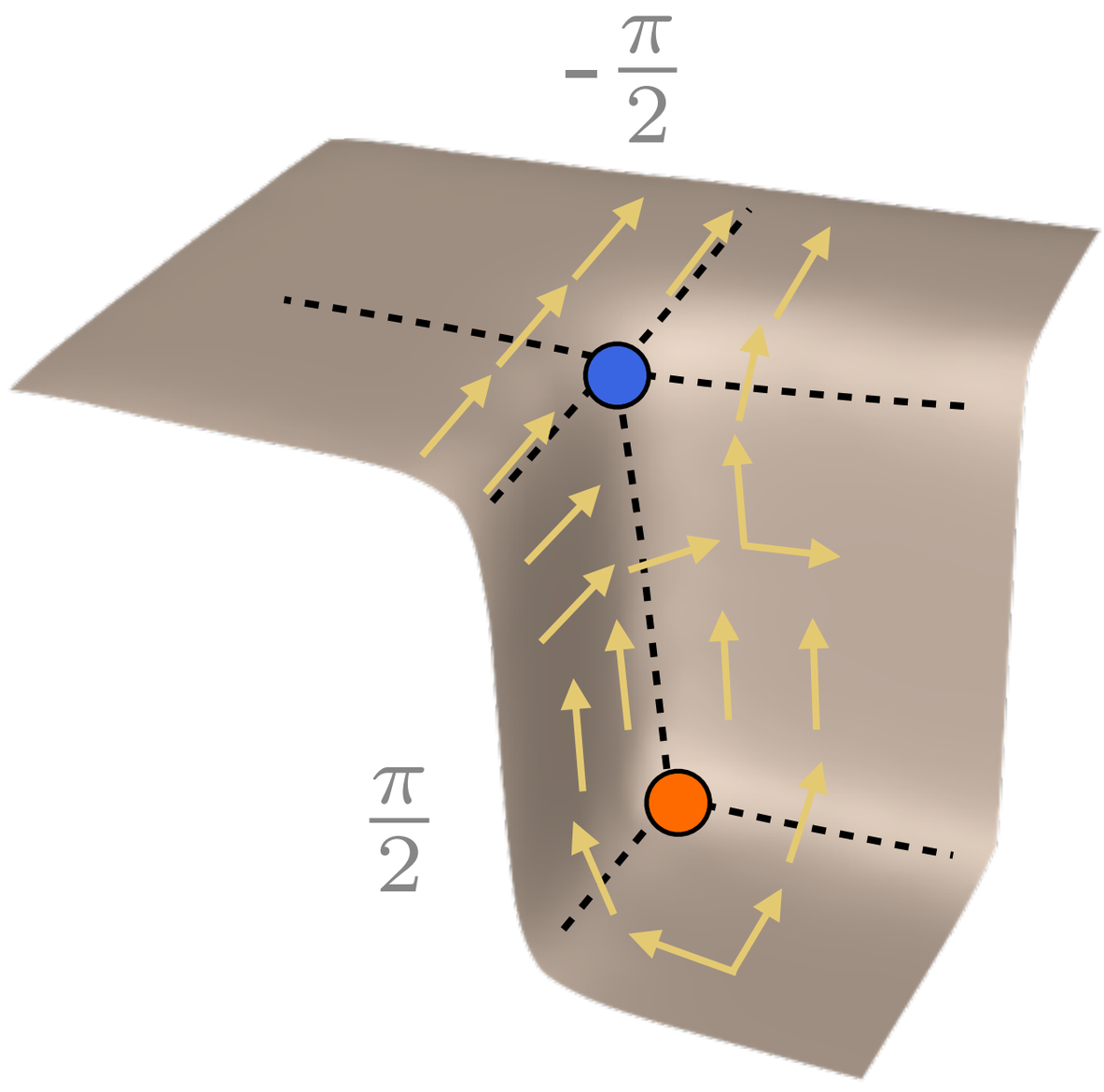




IRREGULAR VERTICES

- construct alternative Gaussian curvature, (close to original)
- by constructing alternative parallel transport





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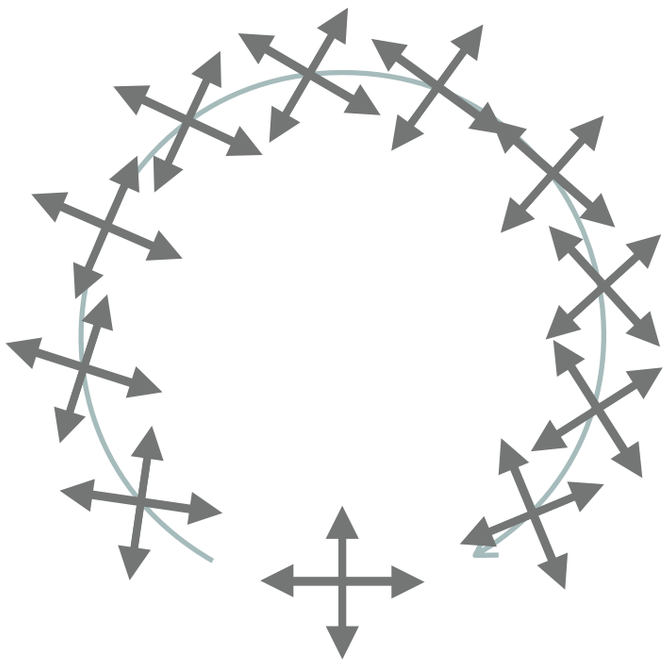

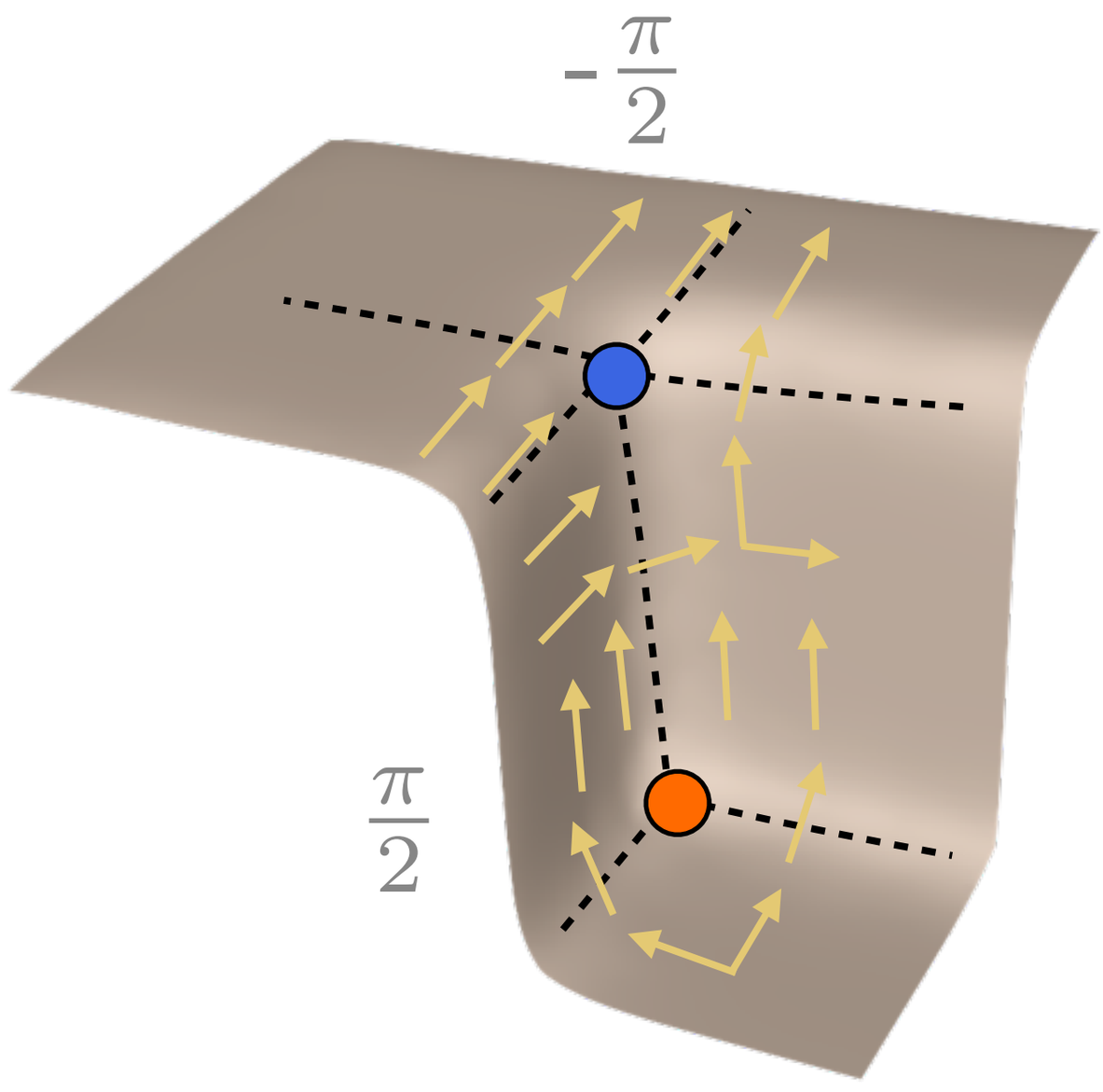

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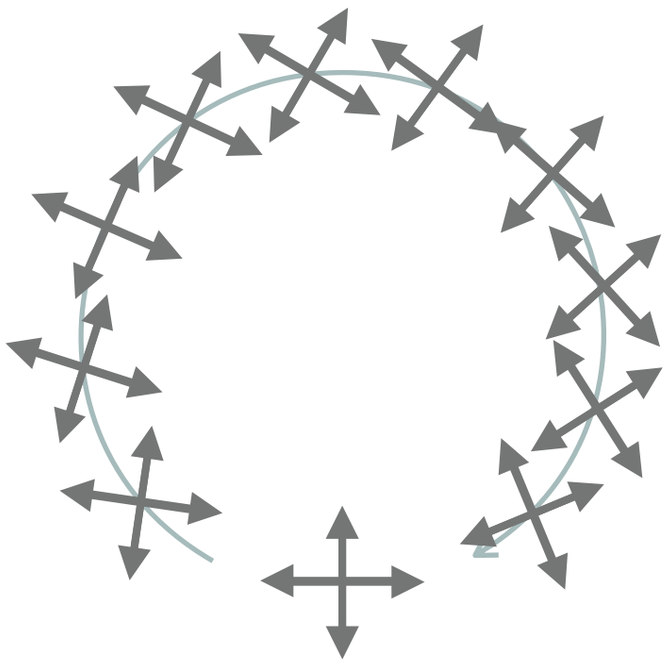
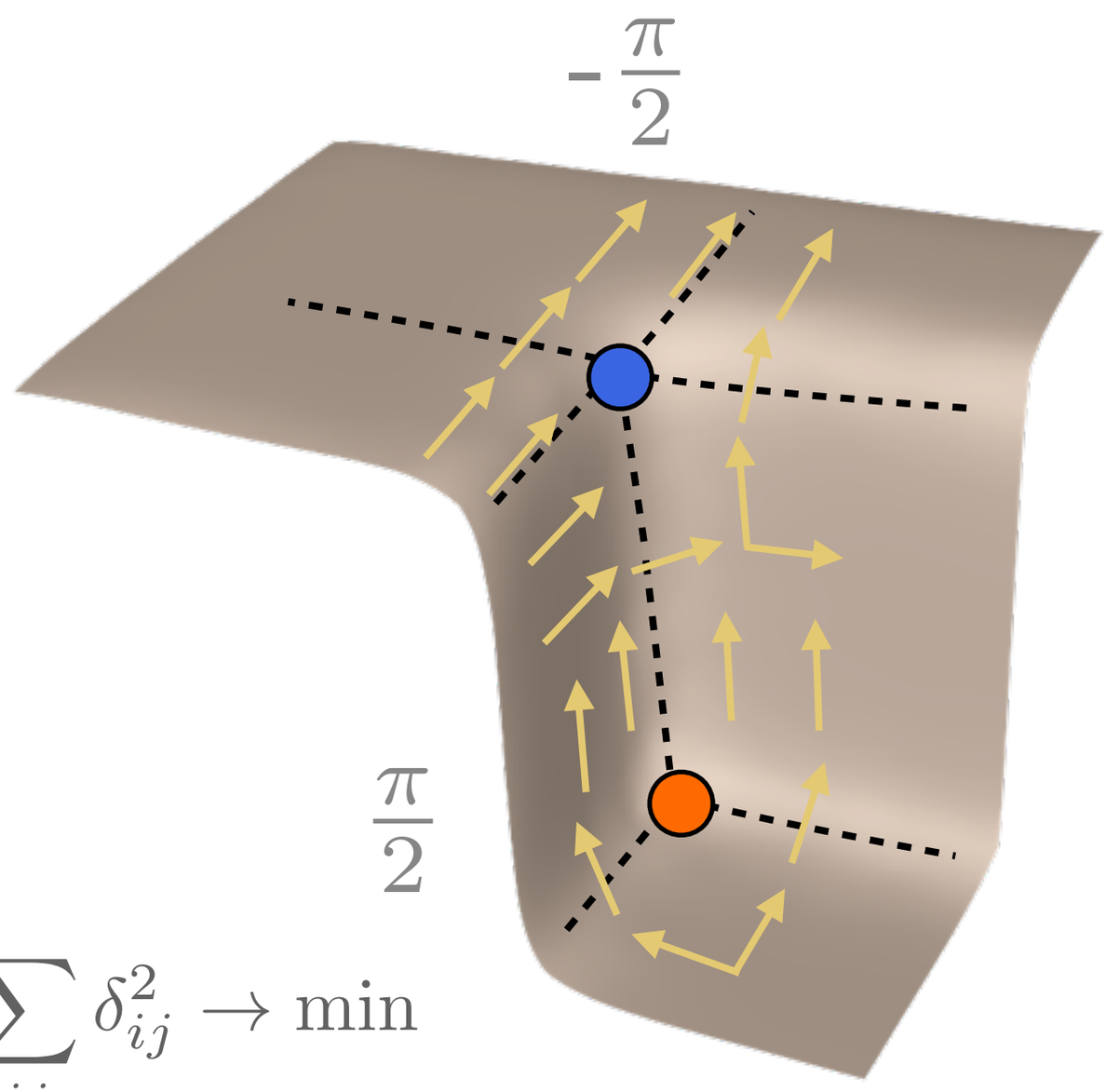


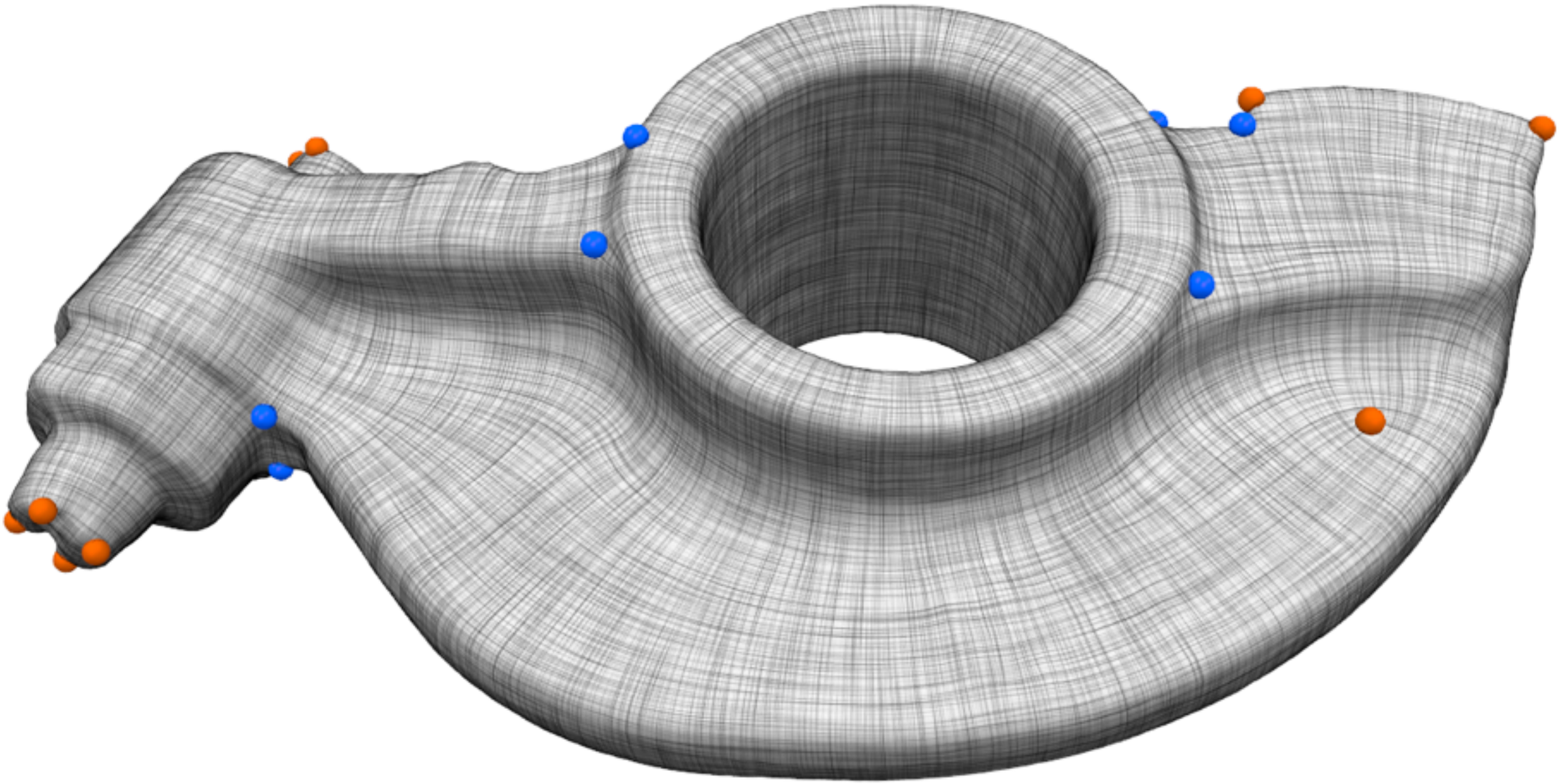


- As-Levi-Civita-as-possible $\sum_{ij} \delta_{ij}^2 \rightarrow \min$

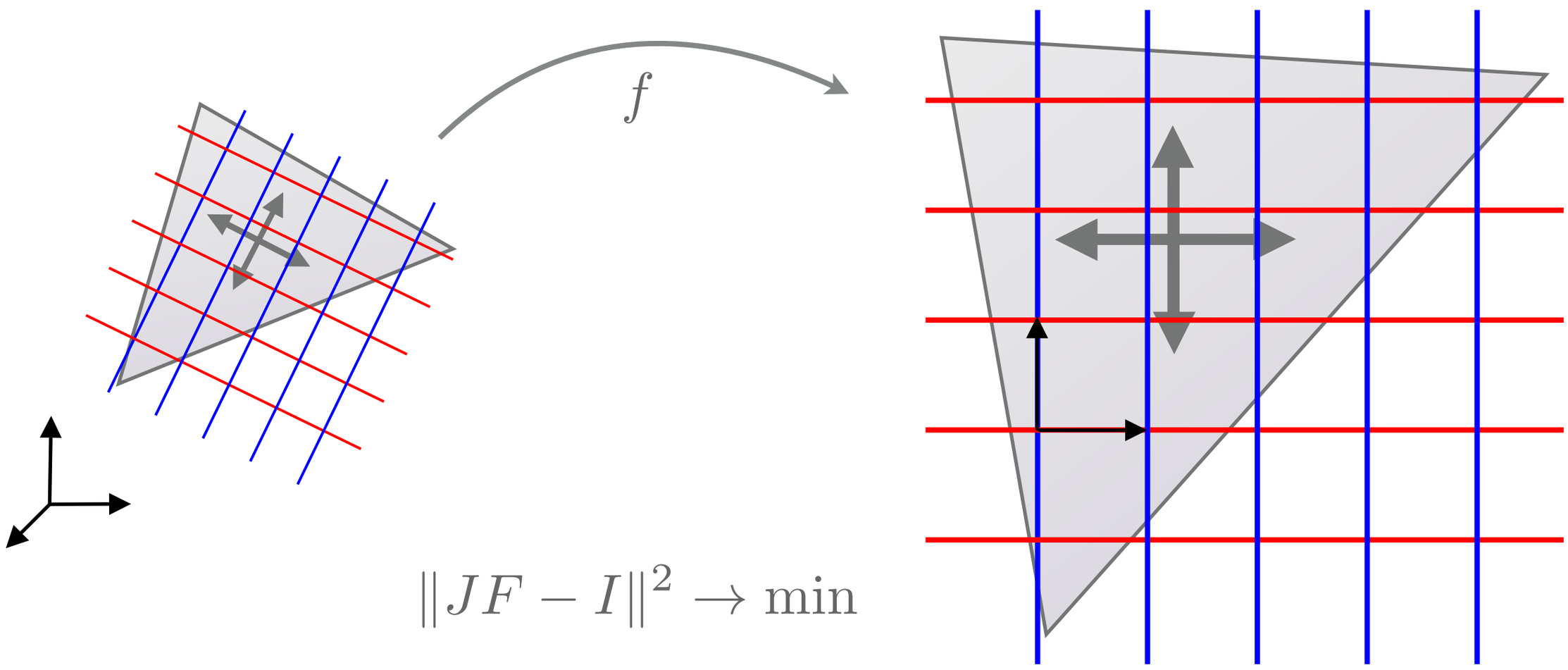
Li et al. 2006 Bommes et al. 2009
Ray et al. 2008 Crane et al. 2010

Vaxman et al. 2016: Directional Field Synthesis

CROSS FIELD



FIELD GUIDED PARAMETRIZATION



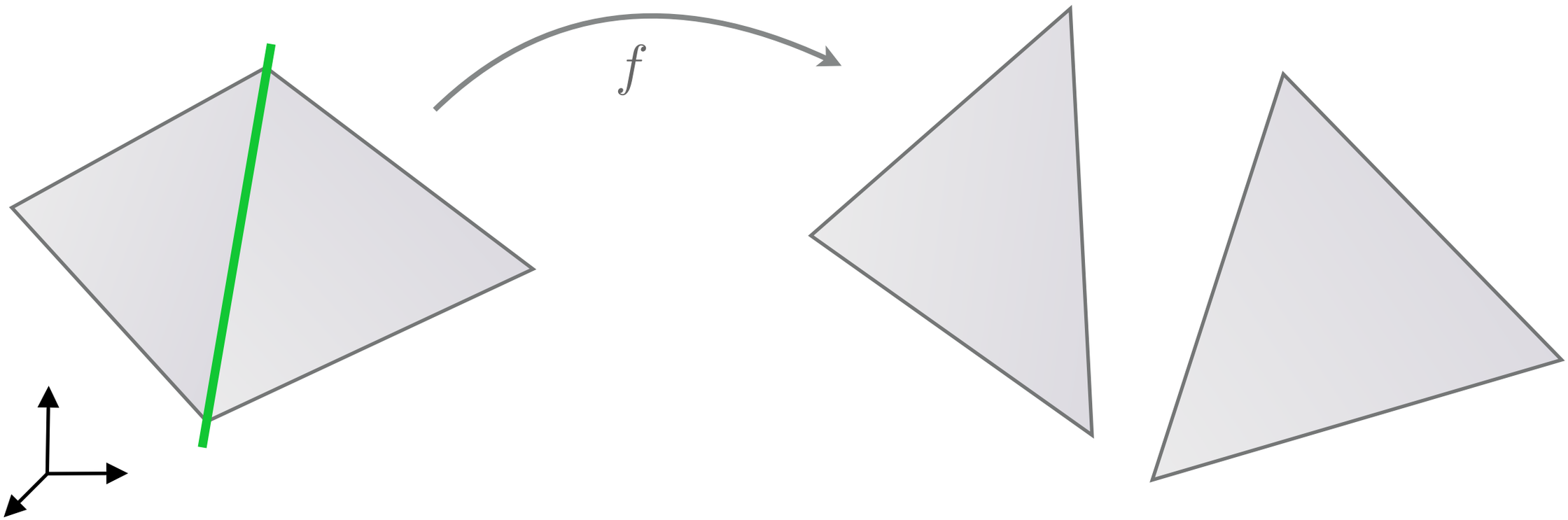
$$\|JF - I\|^2 \to \min$$

$$J = \nabla f$$

$$F = [\vec{u}\ \ \vec{v}]$$

Bommes et al. 2013
Ebke et al. 2014

TRANSITION CONSTRAINTS



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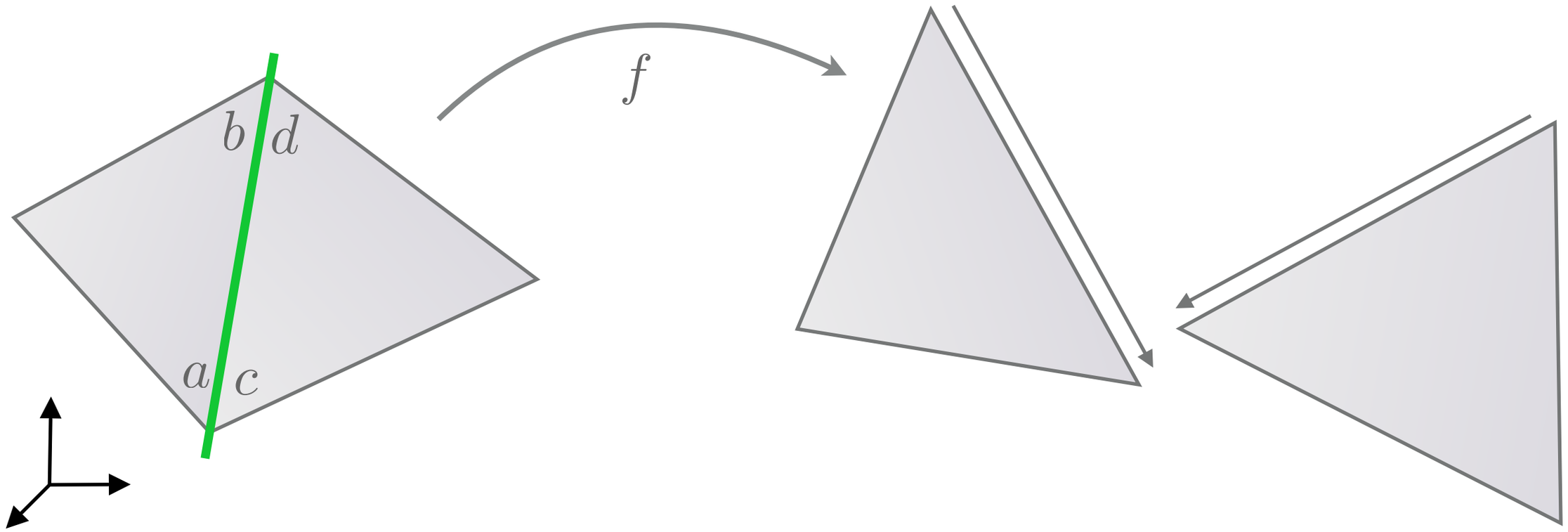
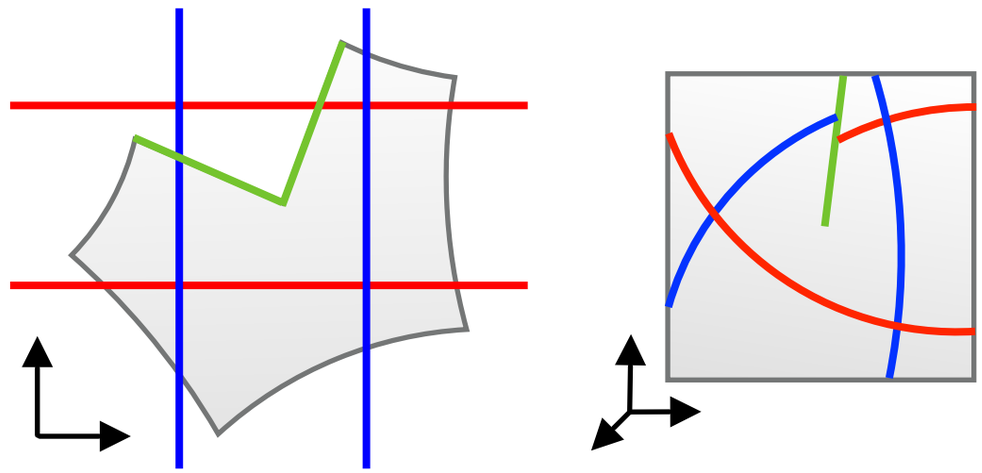
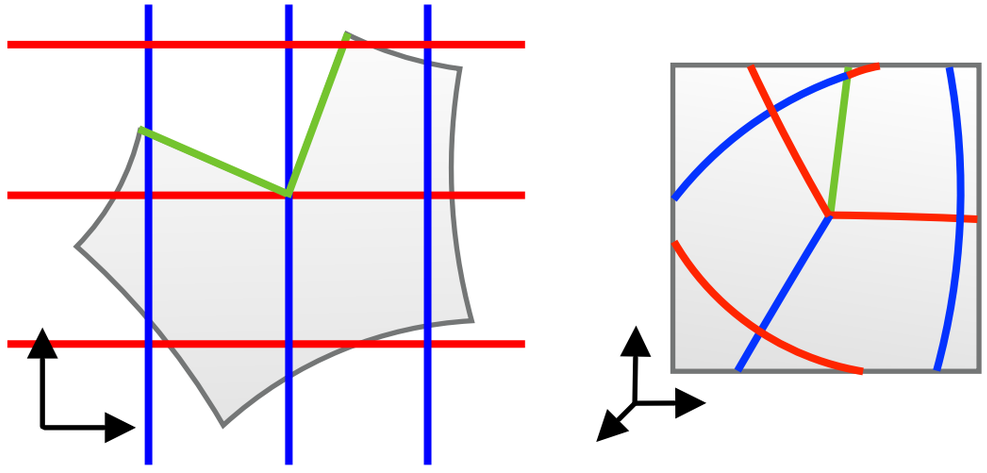


$$R^{90^\circ}\big(f(a) - f(b)\big) = \big(f(c) - f(d)\big)$$

INTEGER CONSTRAINTS

- Some values need to be integer
- But which integers?
  - Solve real-valued
  - Round
  - Solve with constraints
    - all at once *Kälberer et al. 2007*
    - one by one, greedy *Bommes et al. 2009*





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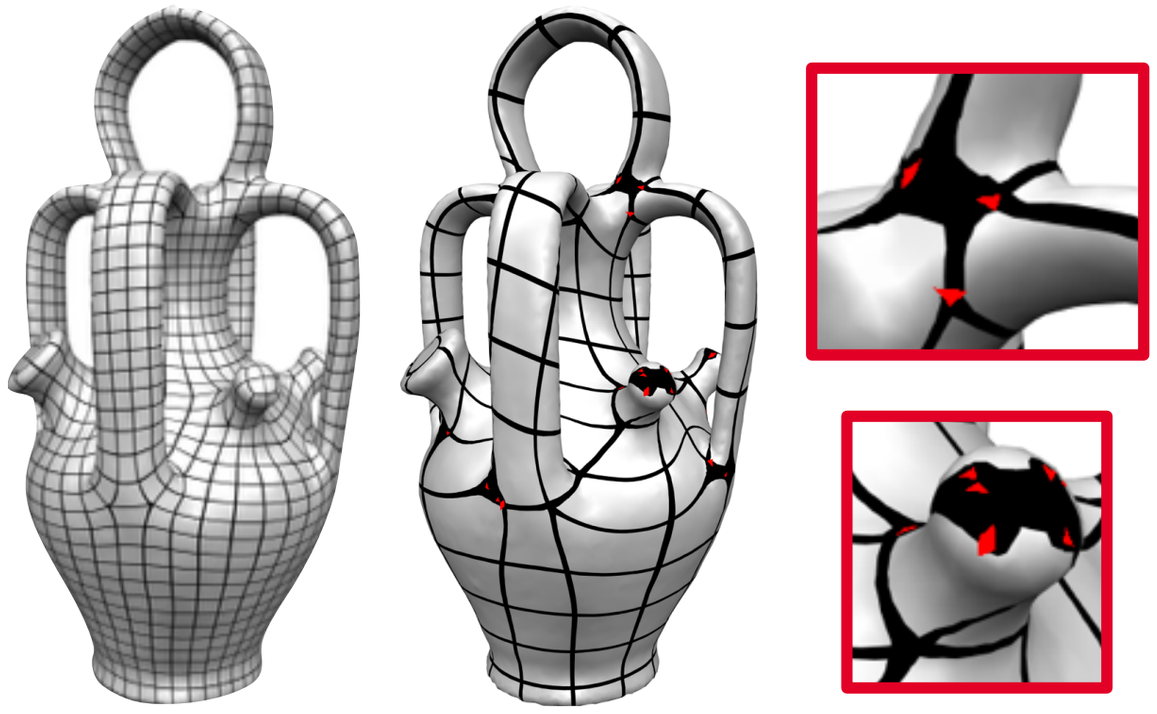
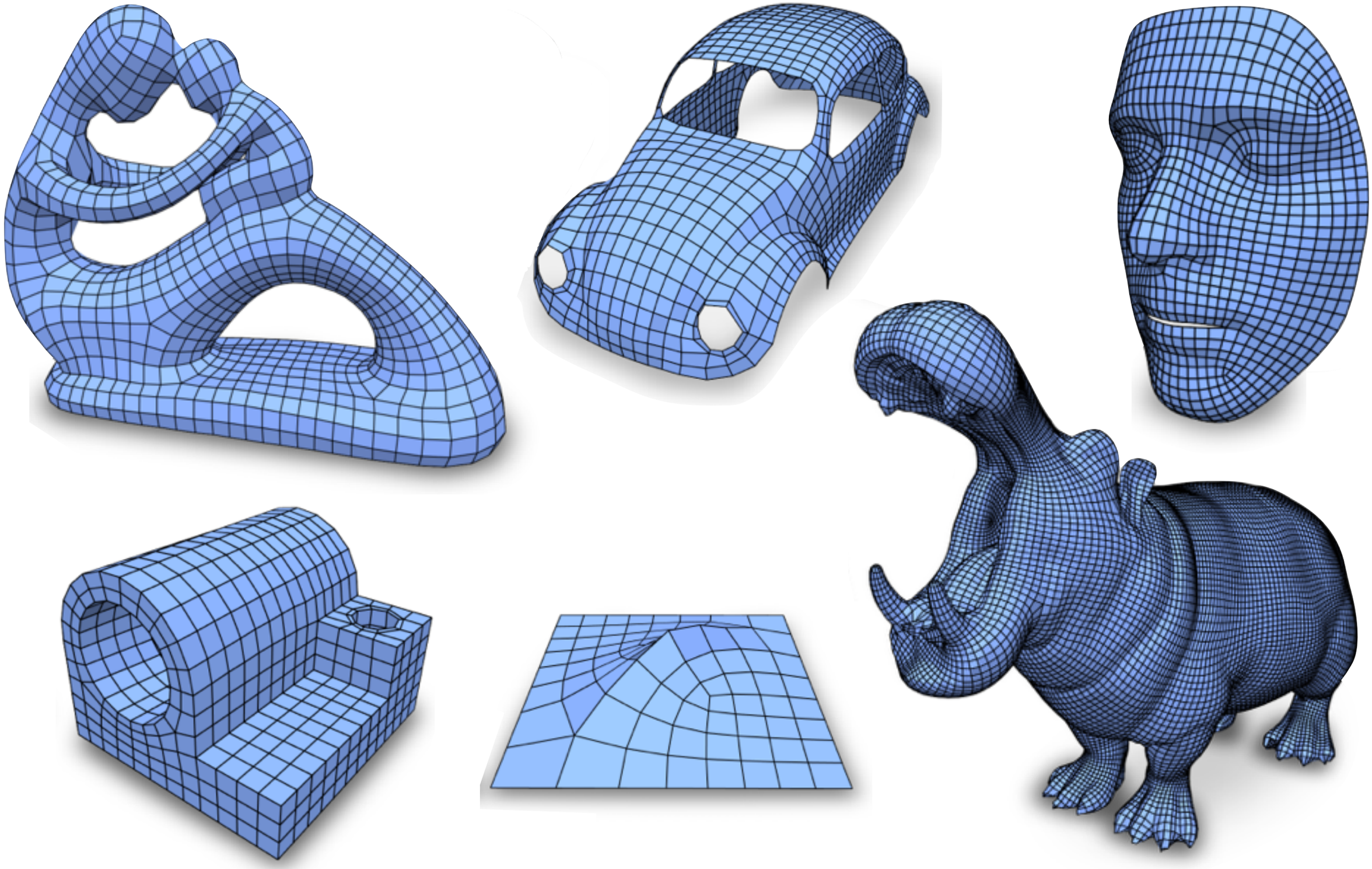


- all at once *Kälberer et al. 2007*
- one by one, greedy *Bommes et al. 2009*
- “try all, take best” *Bommes et al. 2013*
- “differential” approach *Campen et al. 2015*

QUAD MESHES

Campen et al. 2015



ROBUSTNESS

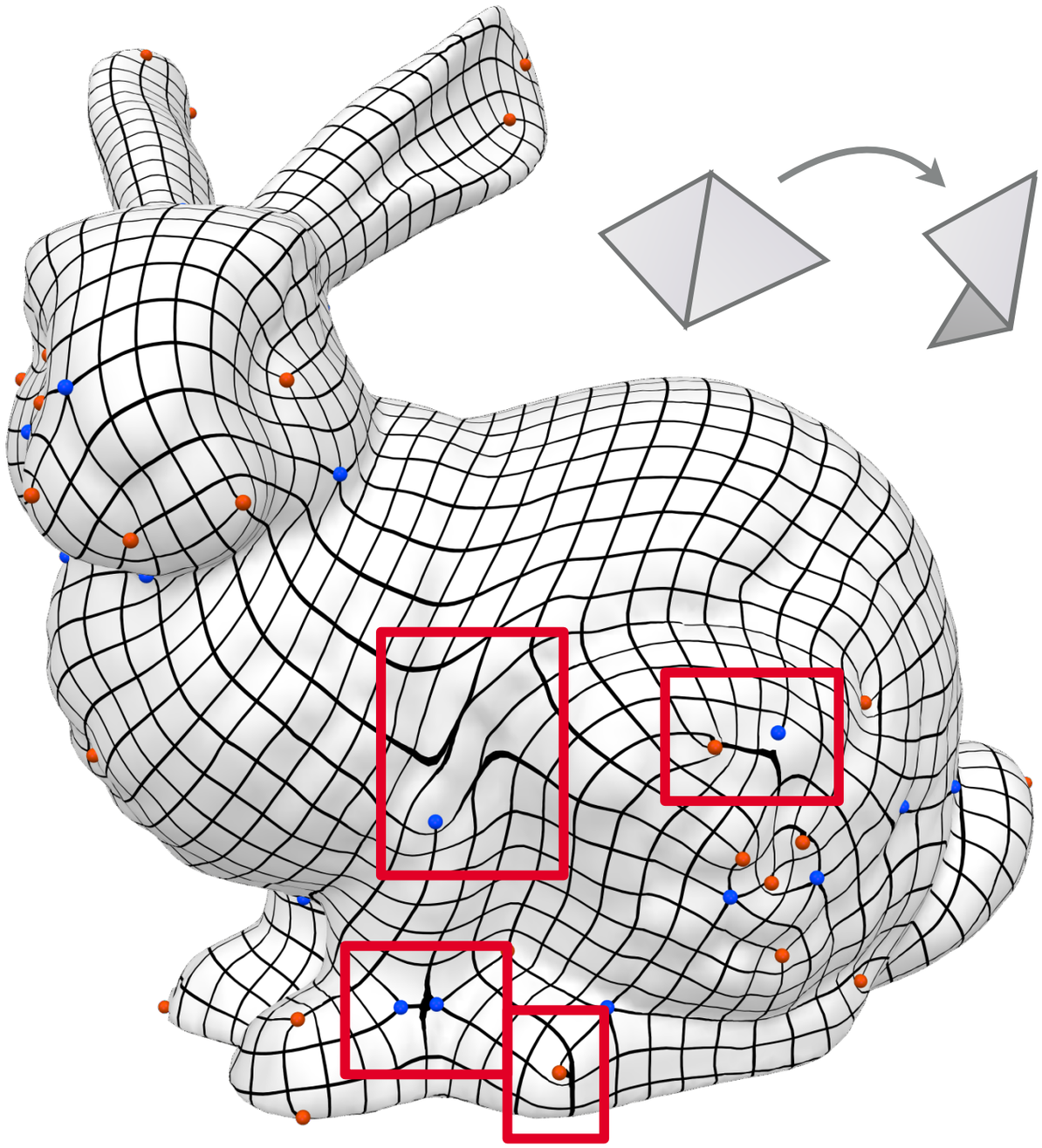
- injectivity

$$\det J > 0$$

$$J = \nabla f$$

Lipman 2012
Bommes et al. 2013
Liu et al. 2016
Myles et al. 2014

Ebke et al. 2013



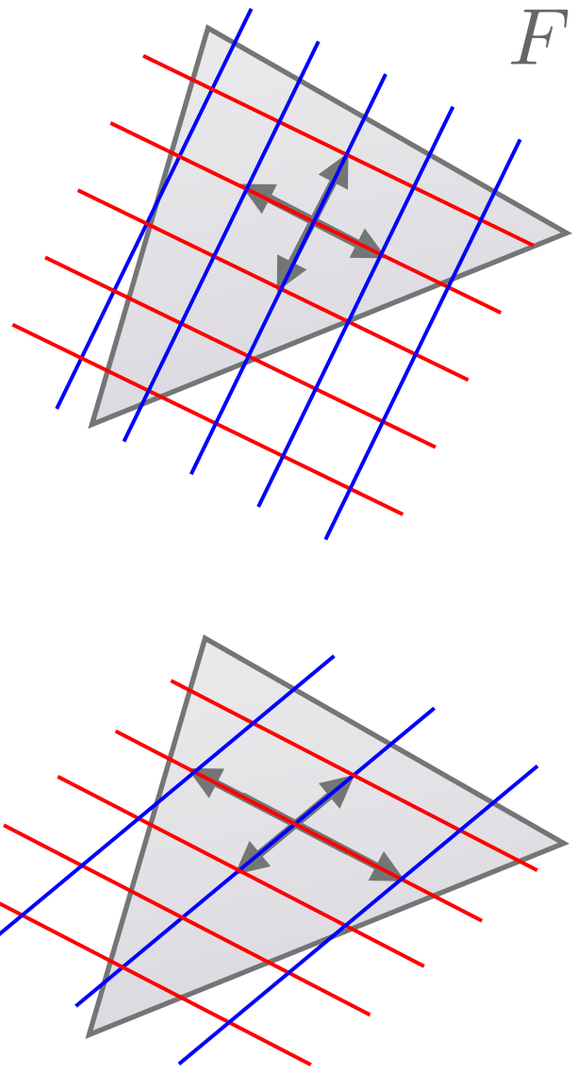
ANISOTROPY & SIZING

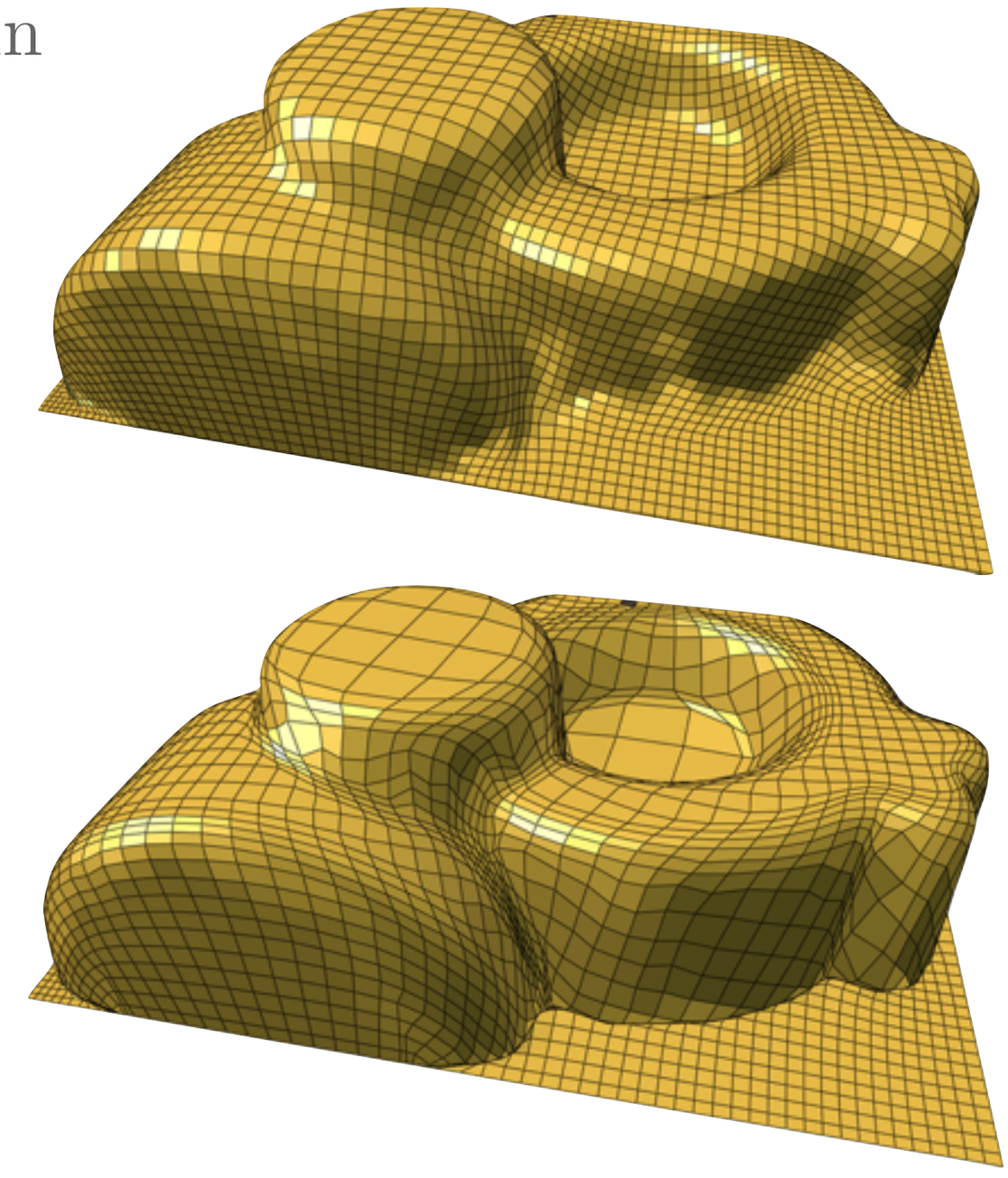
- Use non-orthonormal “cross”-field

Kovacs et al. 2010

$$\|JF - I\|^2 \to \min$$

$$F = [\vec{u}\ \ \vec{v}]$$





ANISOTROPY & SIZING

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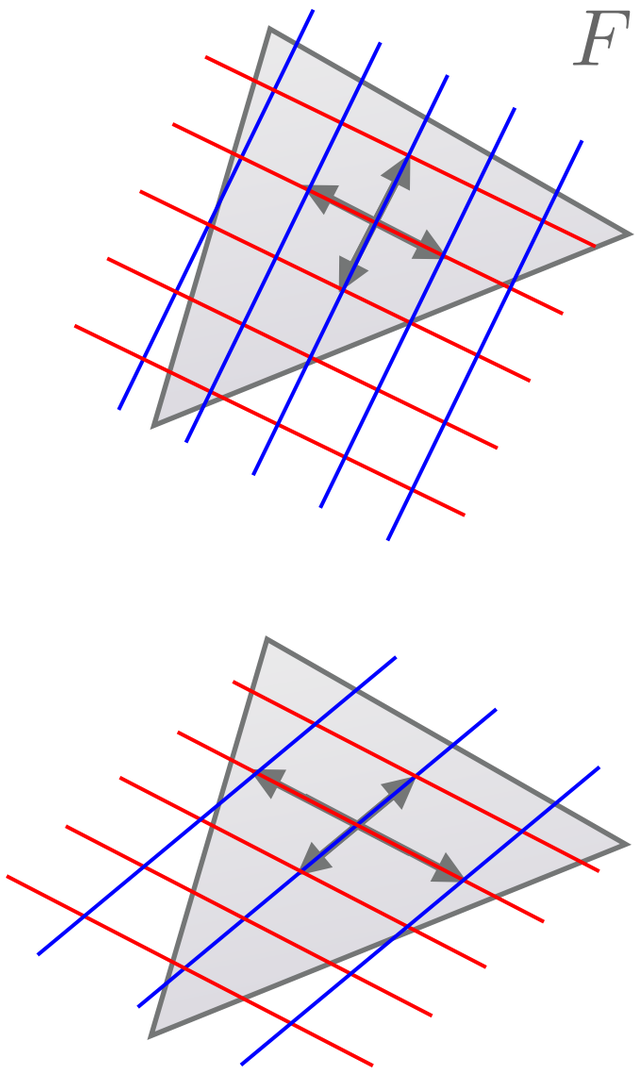
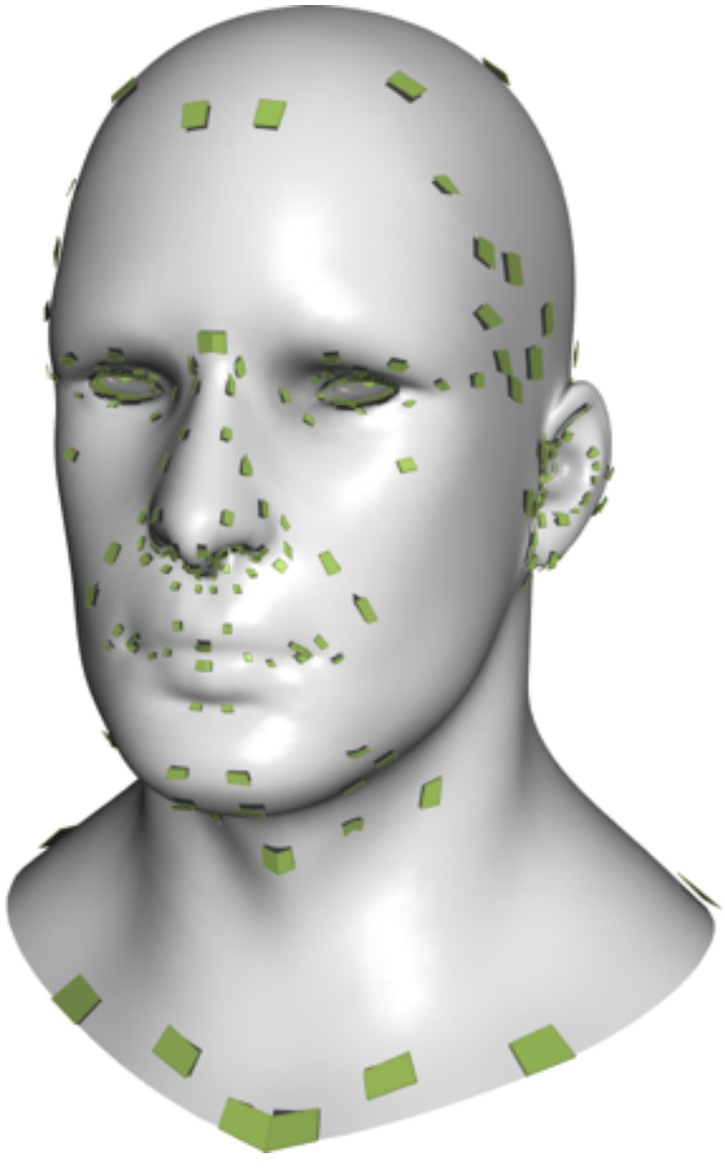
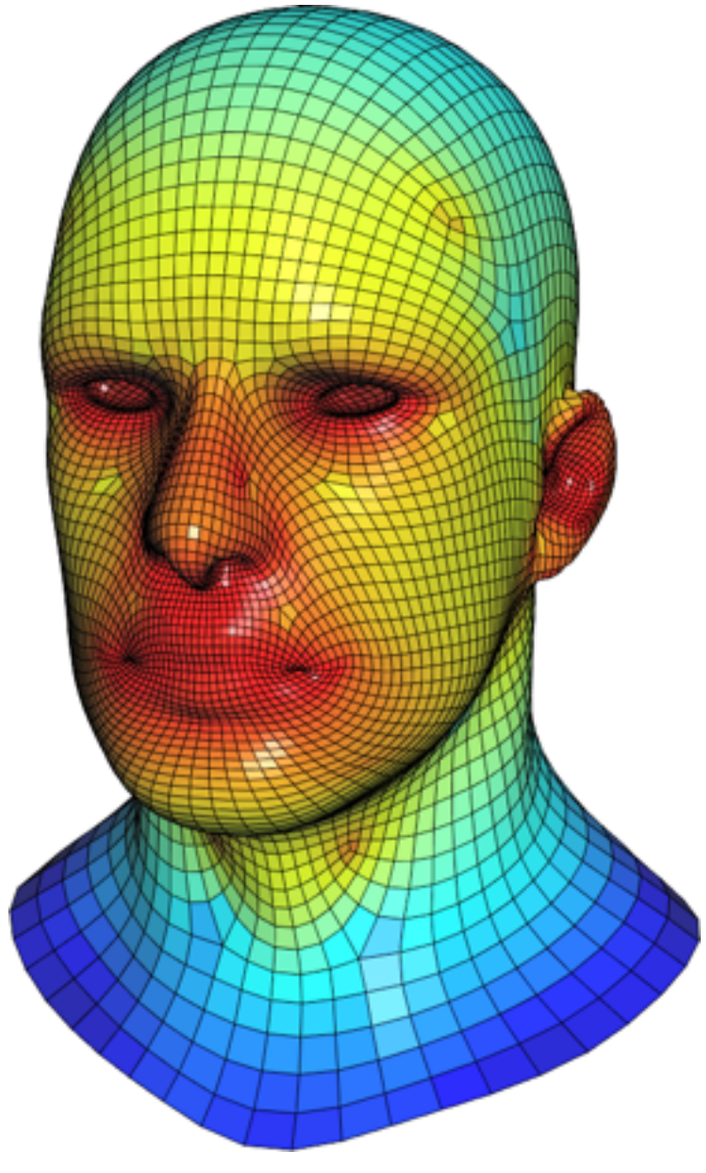
Panozzo et al. 2014

$$\|JF - I\|^2 \rightarrow \min$$

$$F = [\vec{u}\ \ \vec{v}]$$

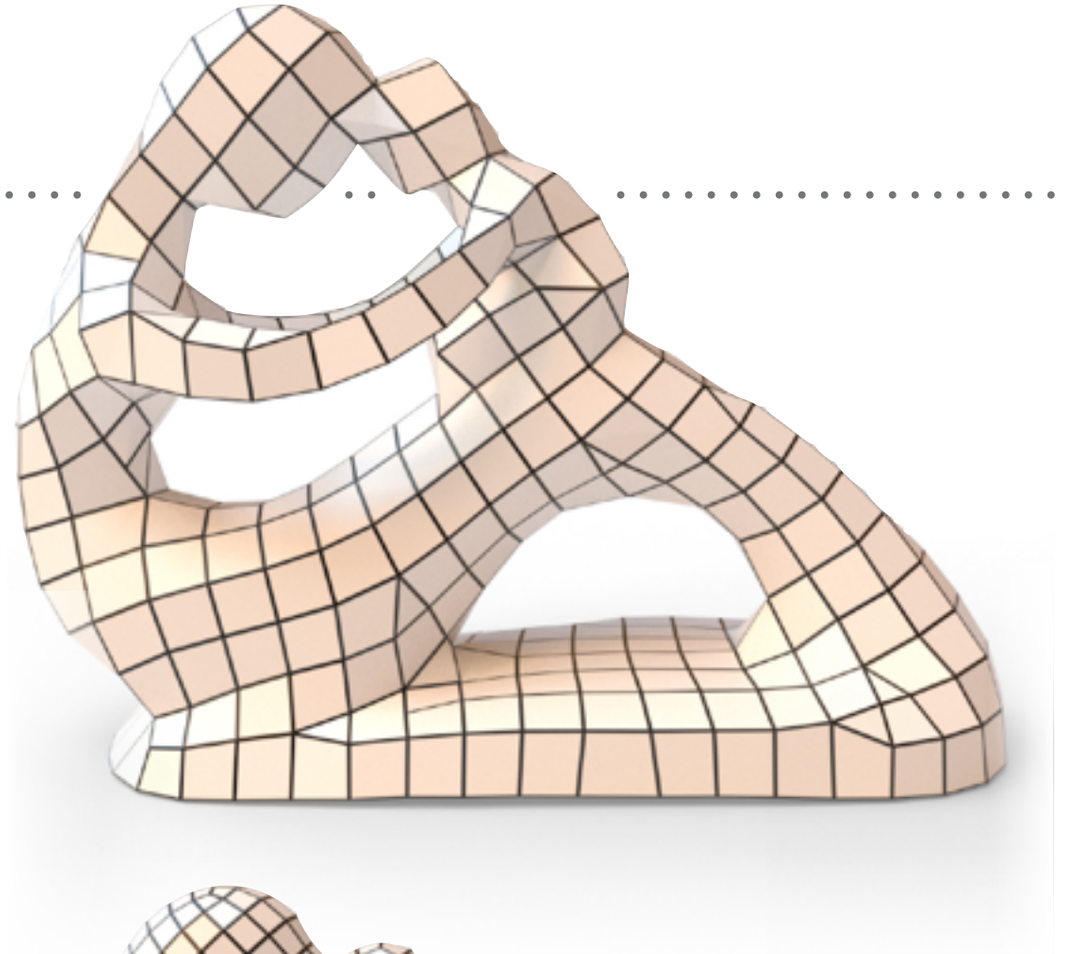


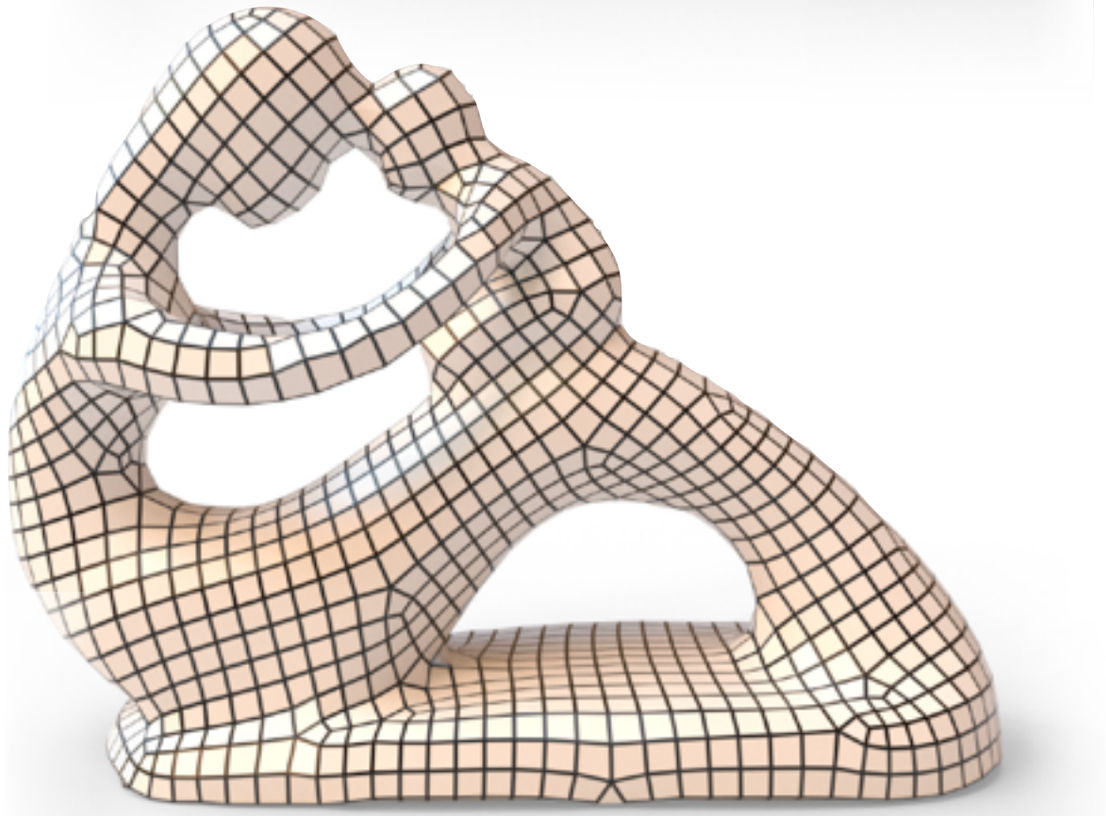




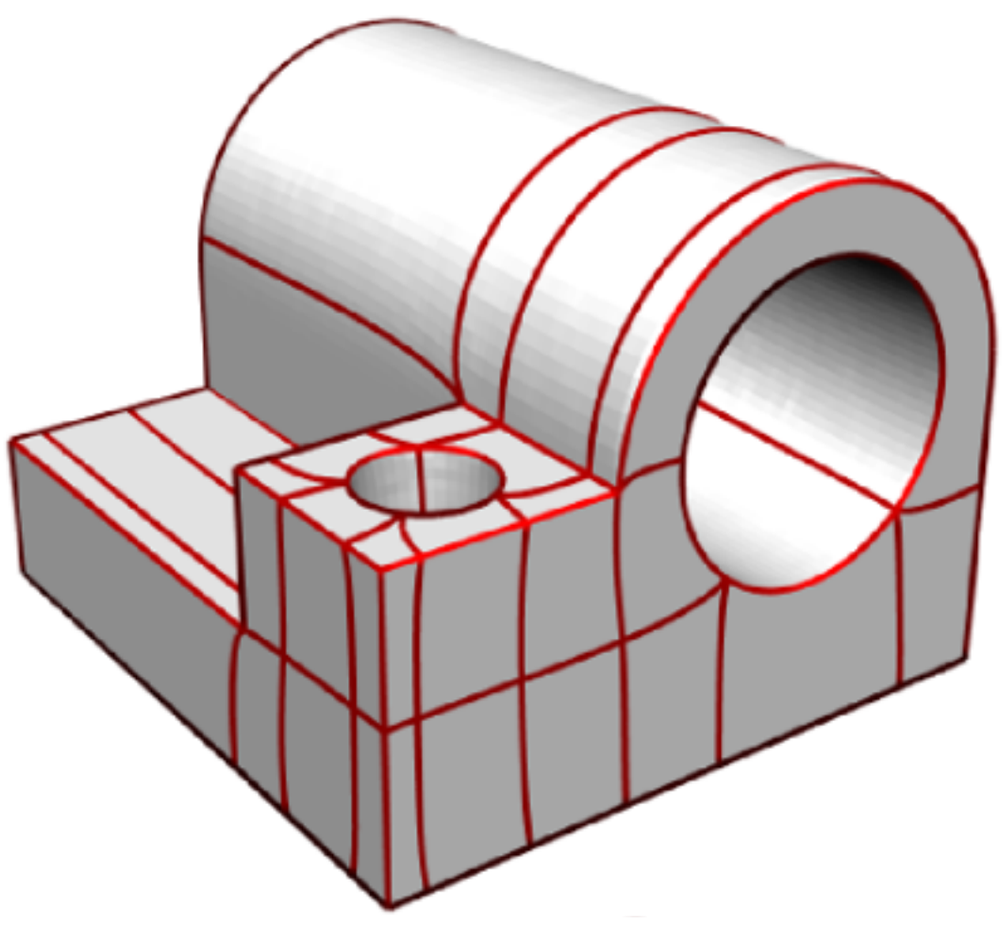
LOCAL PARAMETRIZATION

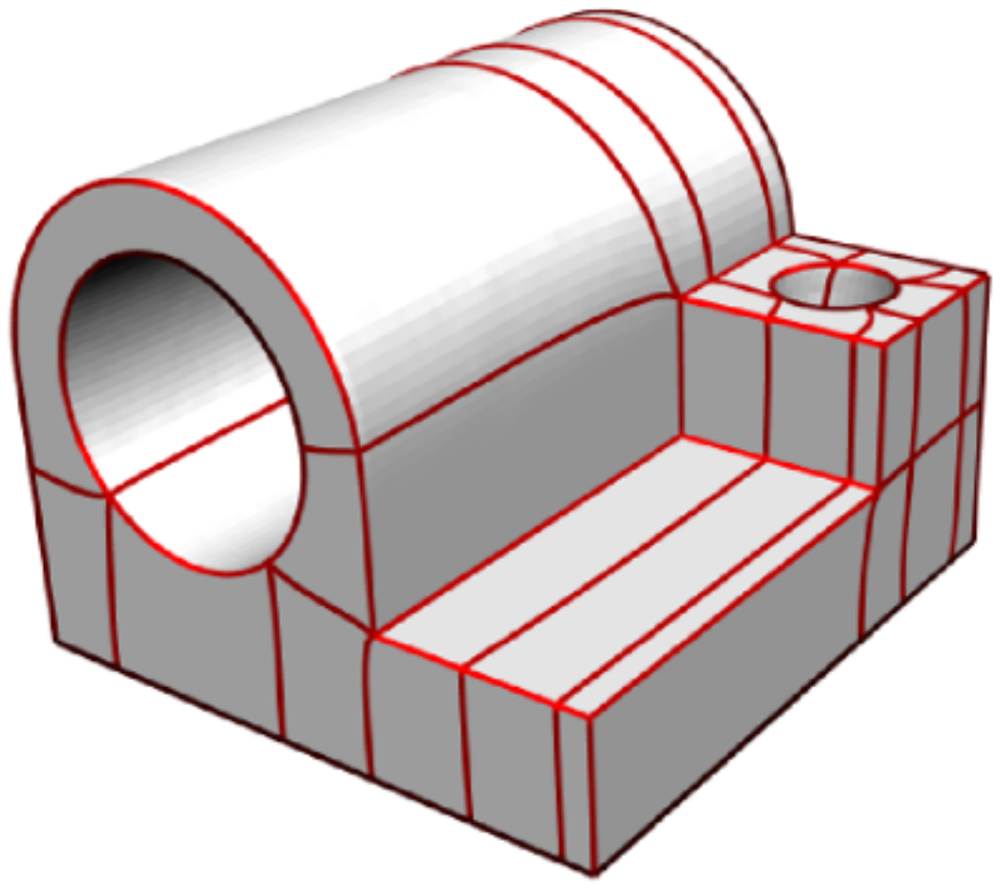
Ray et al. 2006
Knöppel et al. 2015
Jakob et al. 2015





QUAD LAYOUTS

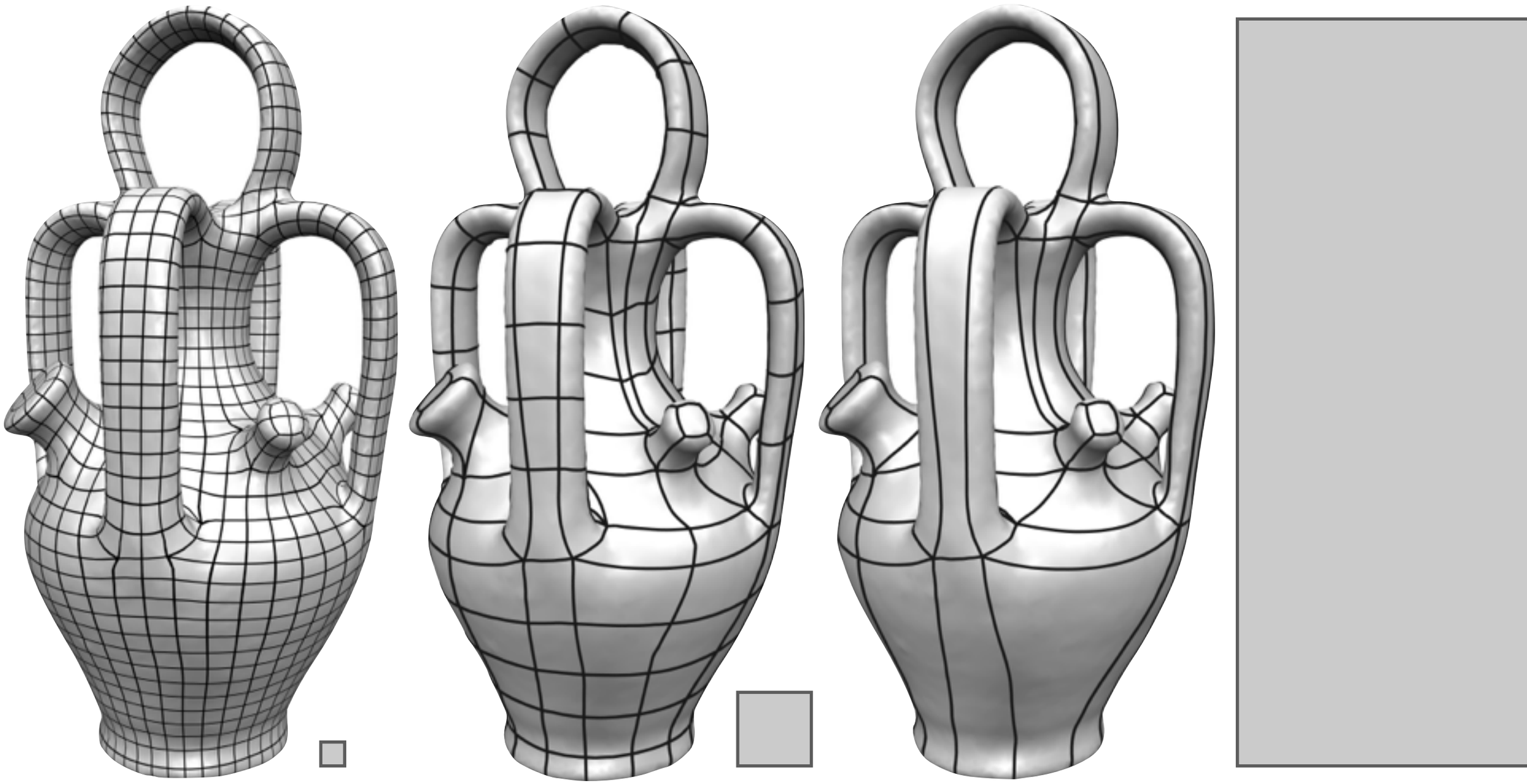




QUAD LAYOUTS

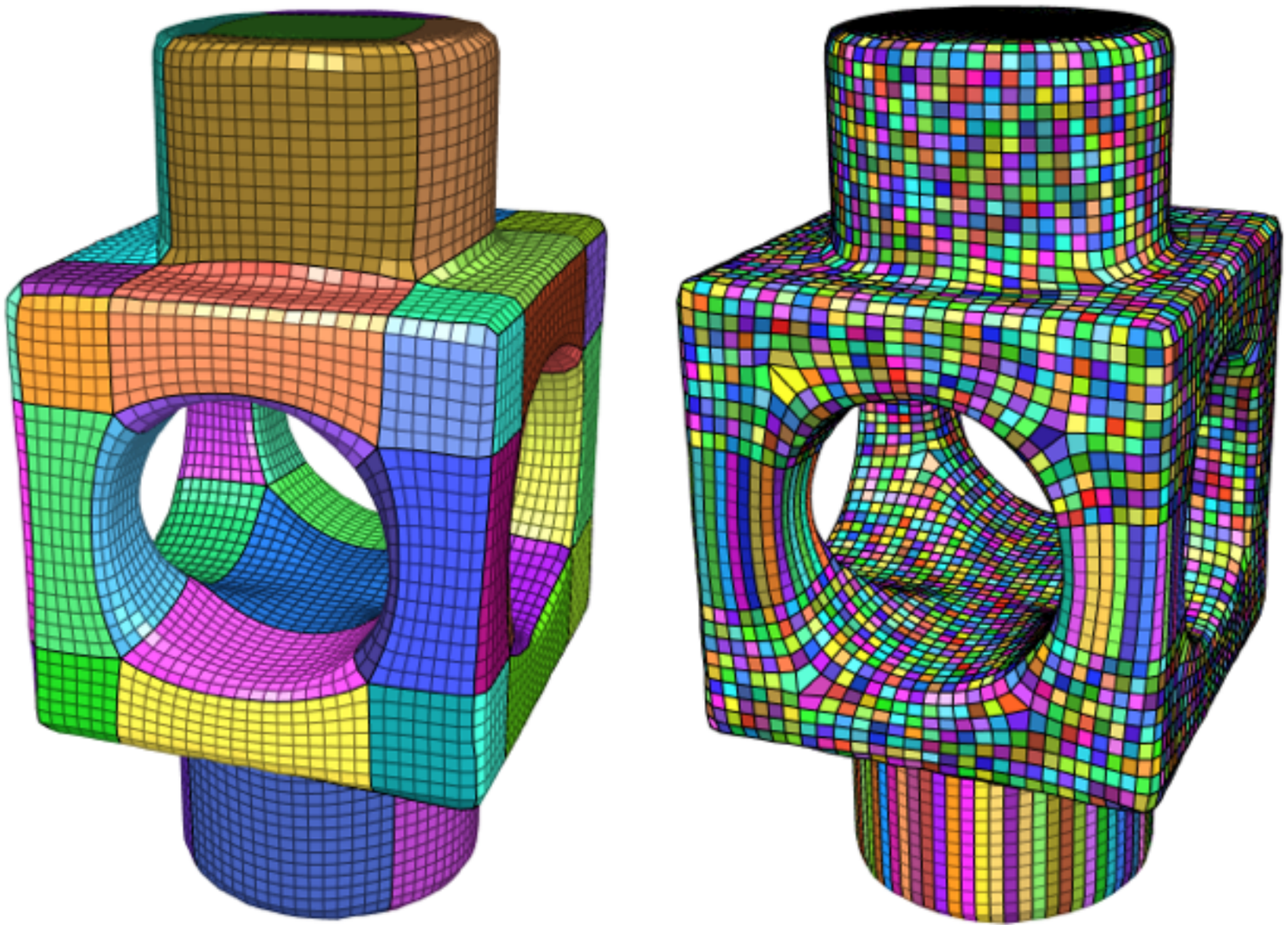
- "Extremal" Integer Grid Maps

Bommes et al. 2013
Campen et al. 2015



QUAD LAYOUTS

- Base-complex

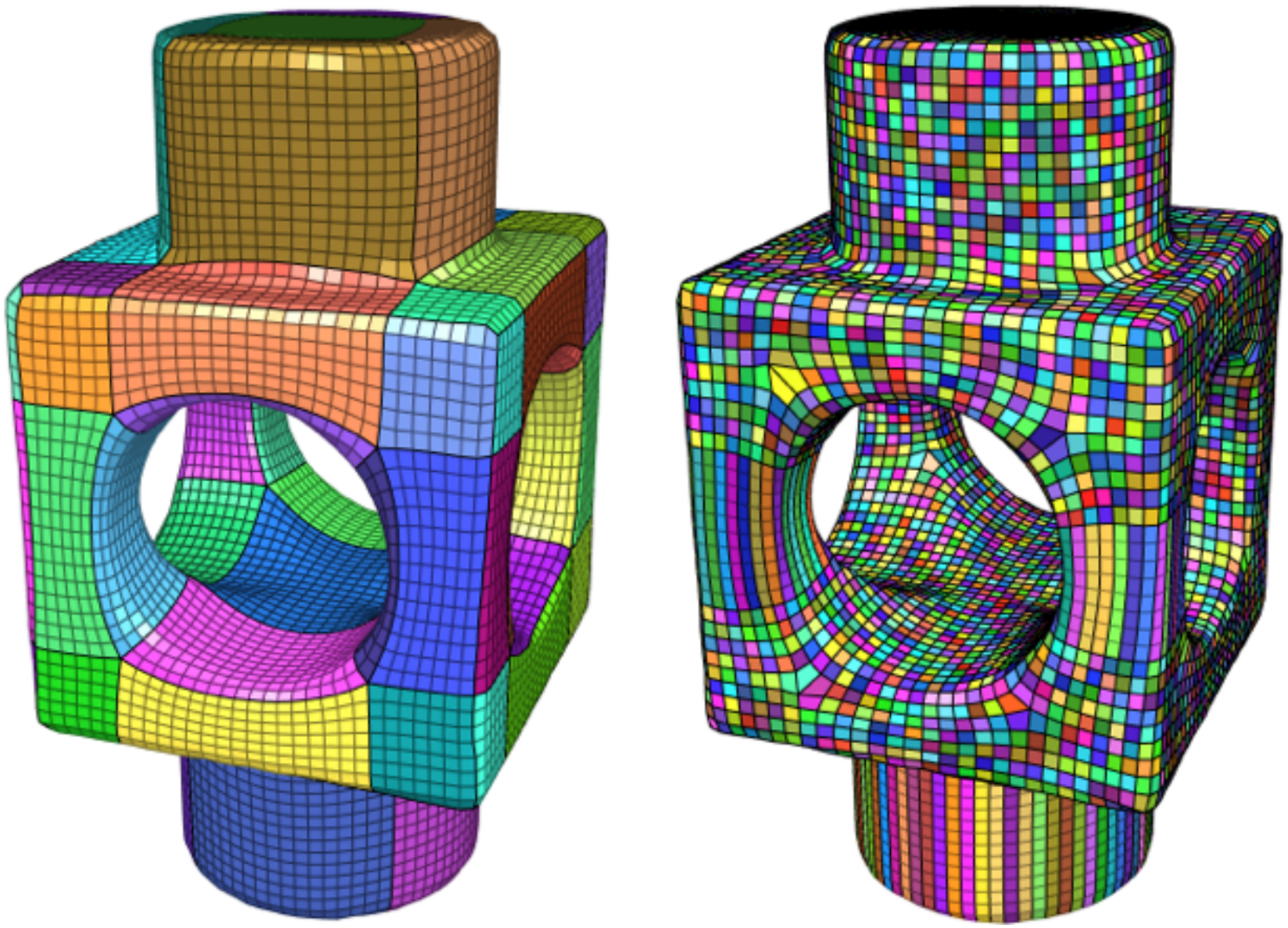


QUAD LAYOUTS

- Base-complex
  - simplify by modifications to quad mesh
  - construct from scratch

Tarini et al. 2011
Bommes et al. 2011

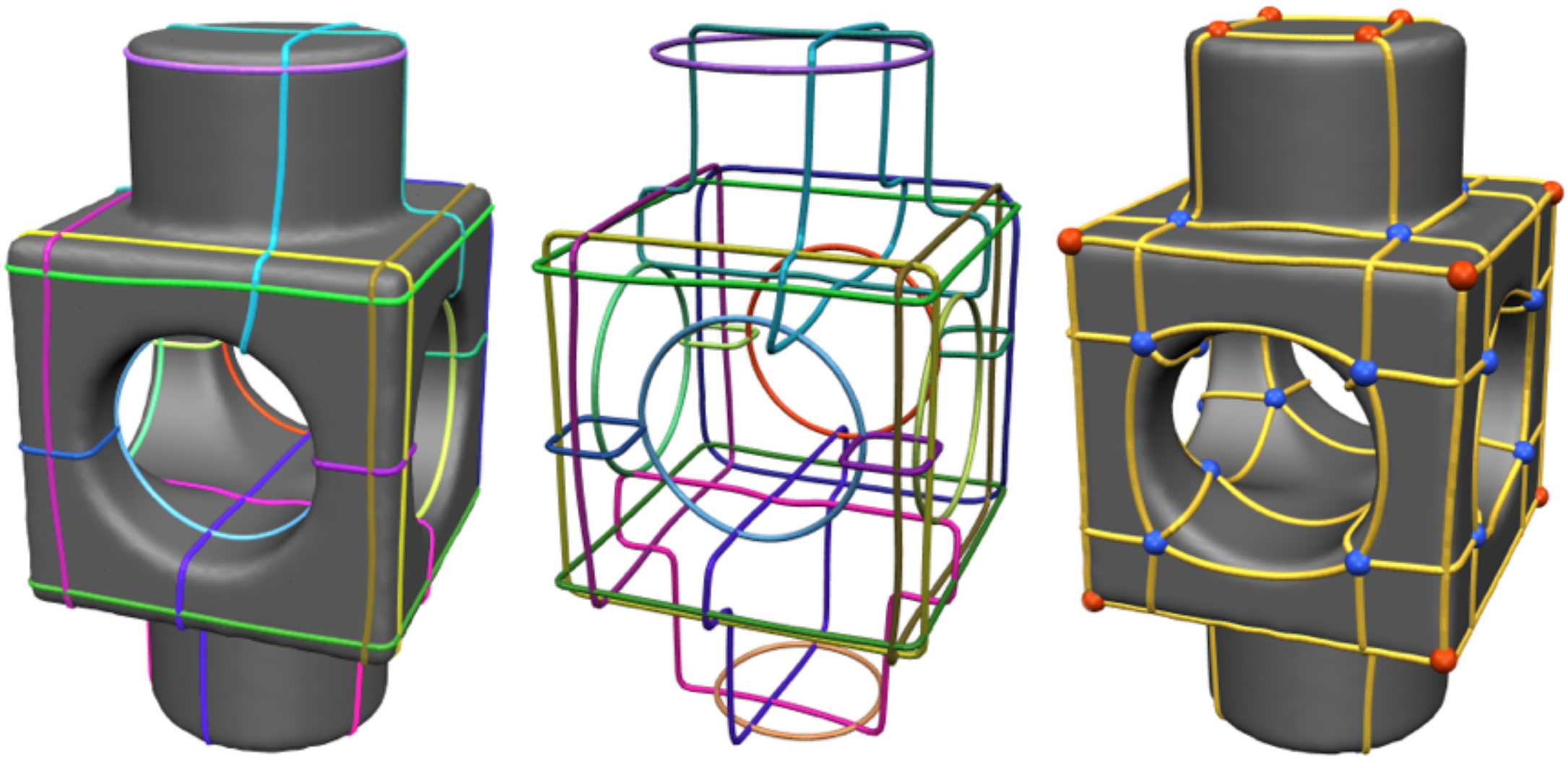
Razafindrazaka et al. 2015
Usai et al. 2015
Campen et al. 2012
Campen & Kobbelt 2014



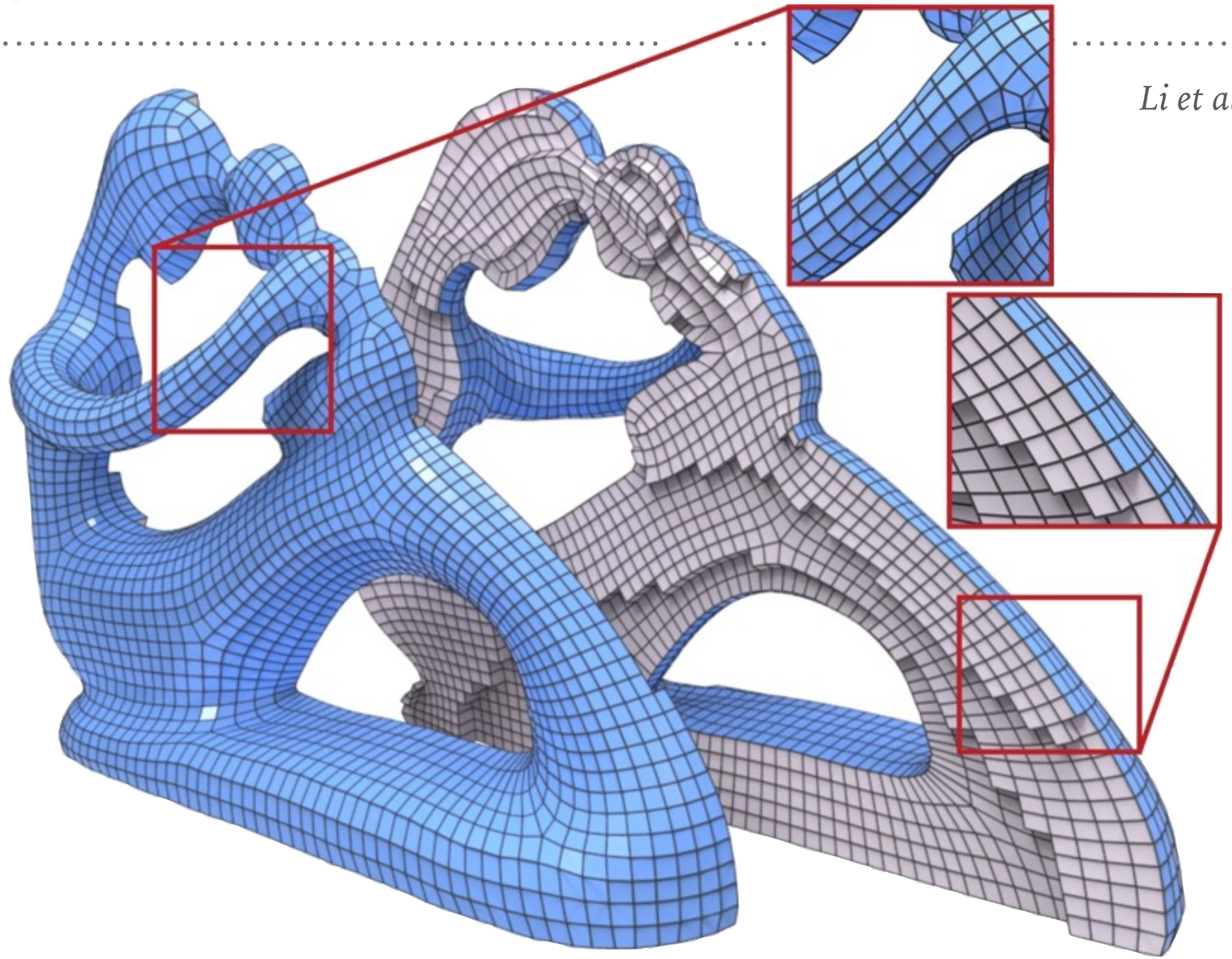
QUAD LAYOUTS: DUAL LOOPS

- Base-complex
  - simplify by modifications to quad mesh
  - construct from scratch

Campen et al. 2012
Campen & Kobbelt 2014



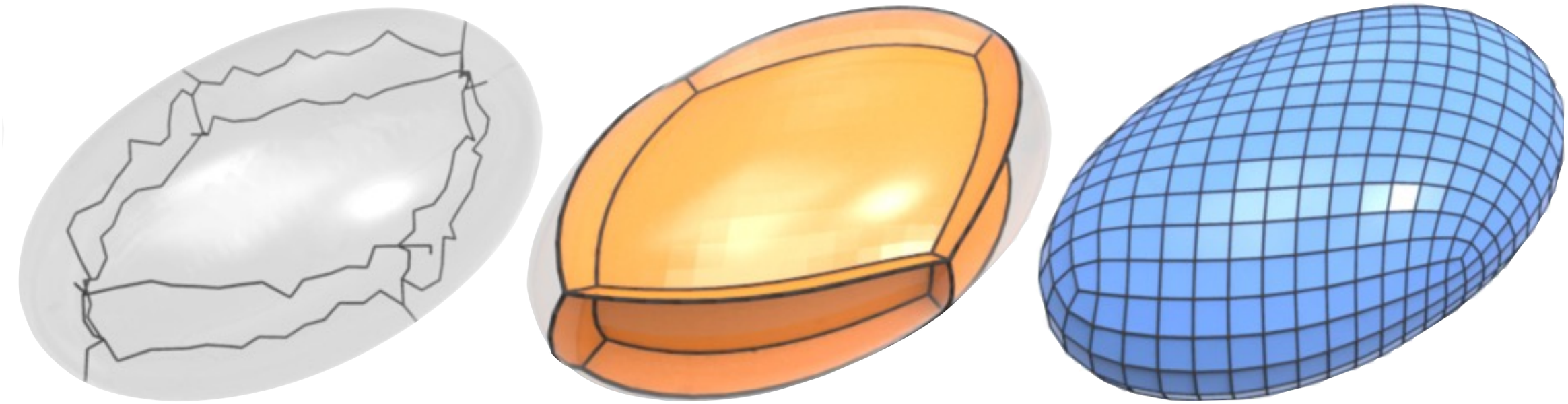
3D?



Li et al. 2012

3D?

Li et al. 2012



3D?

- Param. generalizes $\|JF - I\|^2 \to \min$ $F = [\vec{u}\ \vec{v}]$

3D?

- Param. generalizes $\|JF - I\|^2 \to \min$ $F = [\vec{u}\ \vec{v}\ \vec{w}]$

3D?

- Param. generalizes $\|JF - I\|^2 \to \min \qquad F = [\vec{u}\ \vec{v}\ \vec{w}]$
- Cross fields don't! $\sum \delta_{ij}^2 \to \min$
  - harder to get efficient formulation
  - most frame fields are not "legal"
    - legality conditions global and complex
    - even for the local conditions: no simple way to take them into account (other than by post-processing)

SUMMARY

- **Early Approaches**
  - Pairing, Subdivision, Paving, Tracing
- **Morse-Smale**
  - Eigenfunctions, wave functions
- **Integer Grid Maps**
  - flexible, selective level of control, guiding field
- **Global Structure**
  - Post-processing, Dual Loops, Extremal Integer Grid Maps
- **3D**
  - Main problem: singularity/irregularity structure

RESOURCES

- **libigl** *Jacobson, Panozzo et al.*
  basic implementation of integer grid maps
- **instant-meshes** *Jakob et al.*
  local param. based quad-(dominant) meshing
- **CoMISo** *Bommes, Zimmer et al.*
  mixed integer solver for cross fields, param.
- **QEx, HexEx** *Ebke et al., Lyon et al.*
  robust extraction of meshes from parametrizations

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- Kovacs et al. 2010: *Anisotropic Quadrangulation*
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